

### 计算系统生物学

王勇

#### 中国科学院数学与系统科学研究院



http://zhangroup.aporc.org Chinese Academy of Sciences





#### Gene Regulatory Network Inference

In Systems Biology Framework

#### Yong Wang

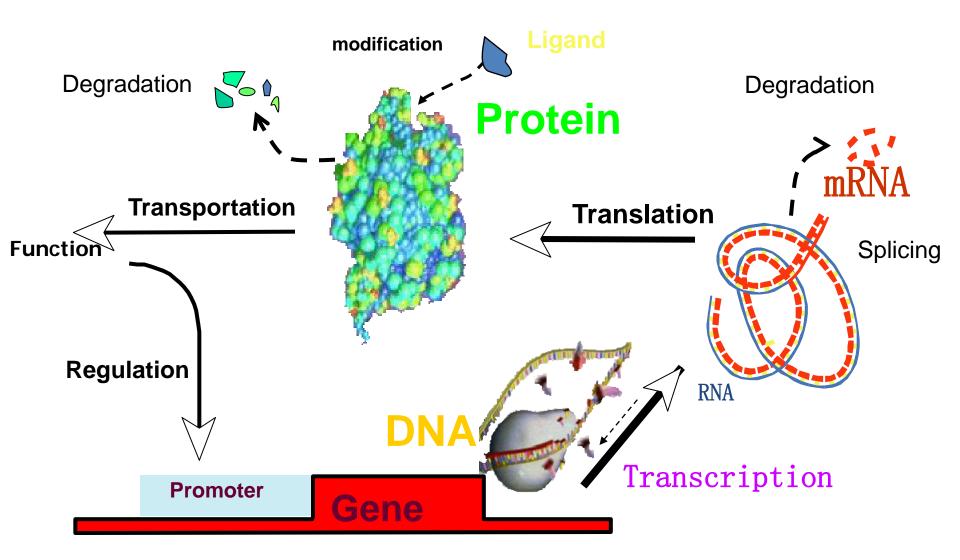
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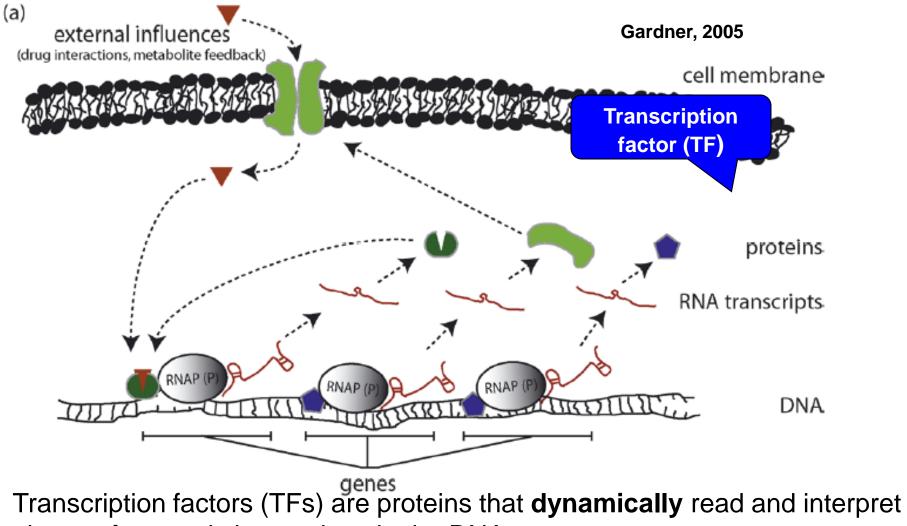


#### Central dogma of molecular biology





#### Gene regulation

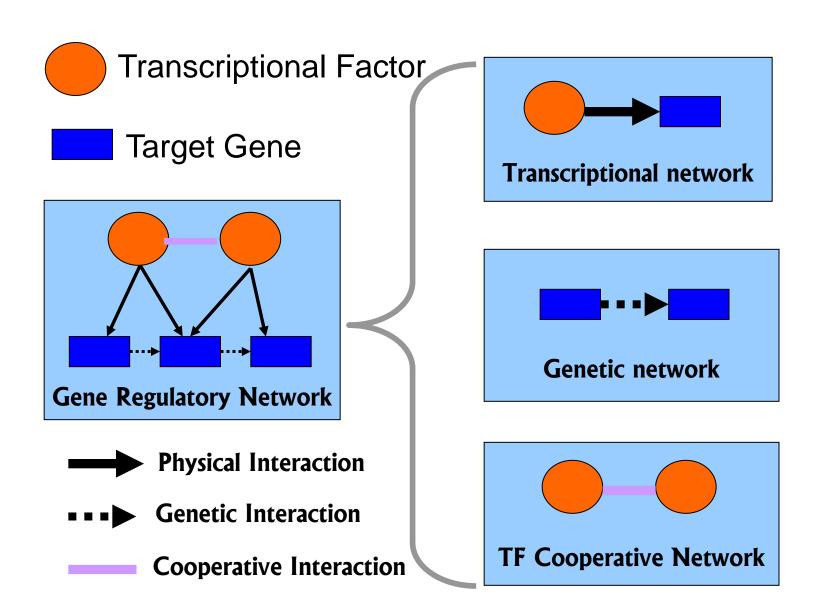


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the static genetic instructions in the DNA

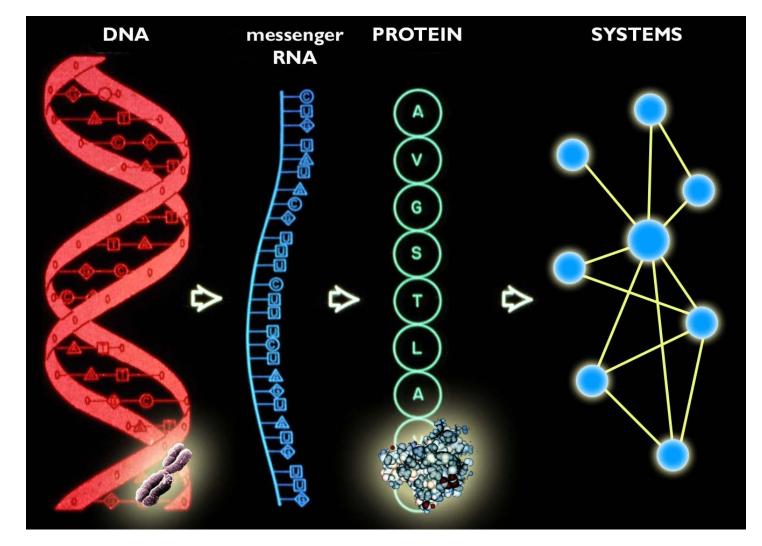


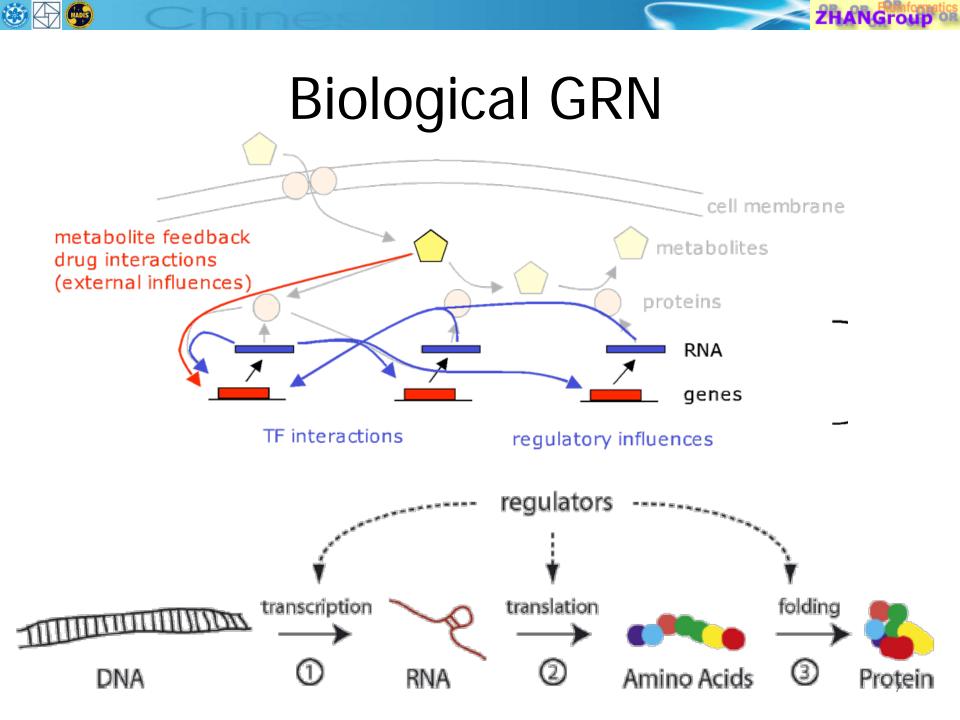
Basic building blocks for gene regulatory network



## Background---GRN

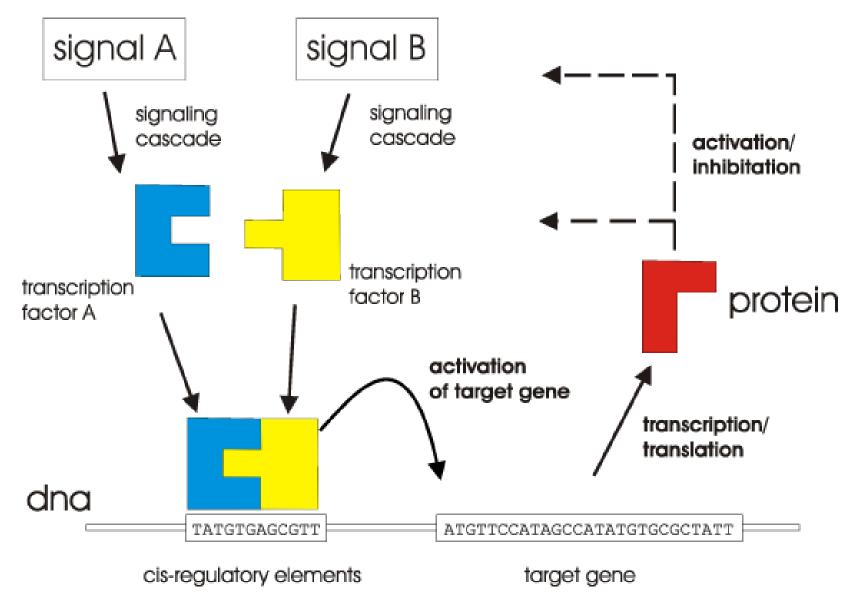
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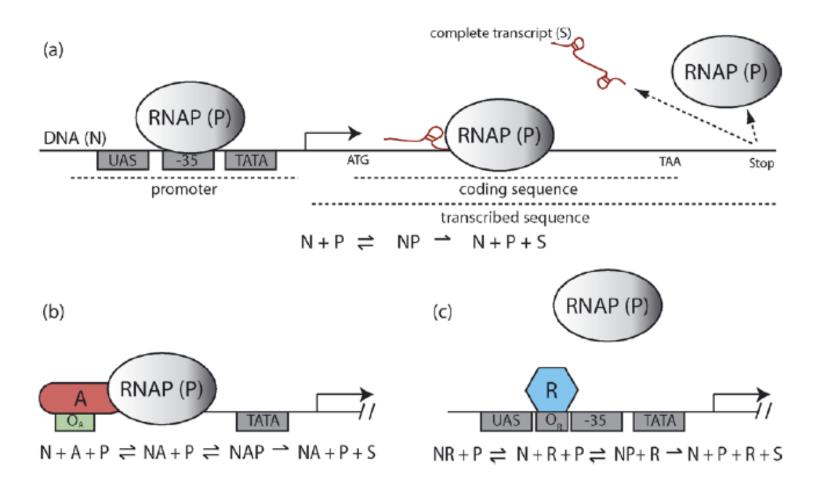




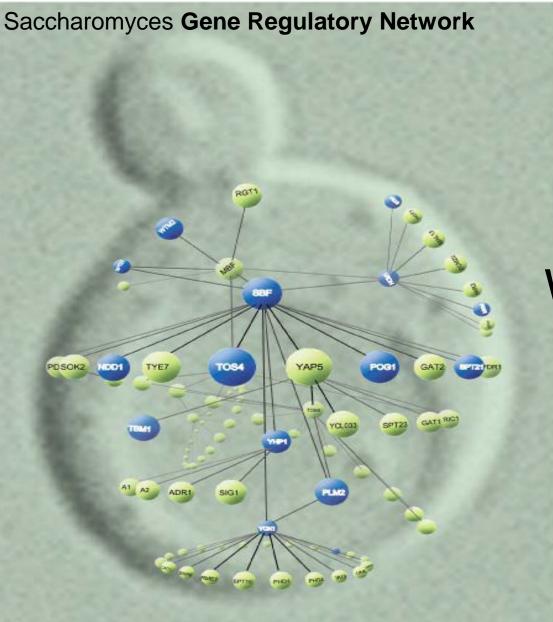


#### ZHANGroup







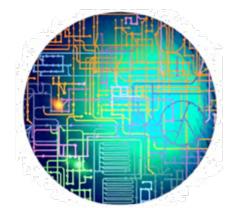


#### What we want?

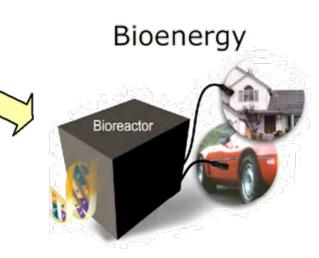


#### Network Inference, Analysis and Control



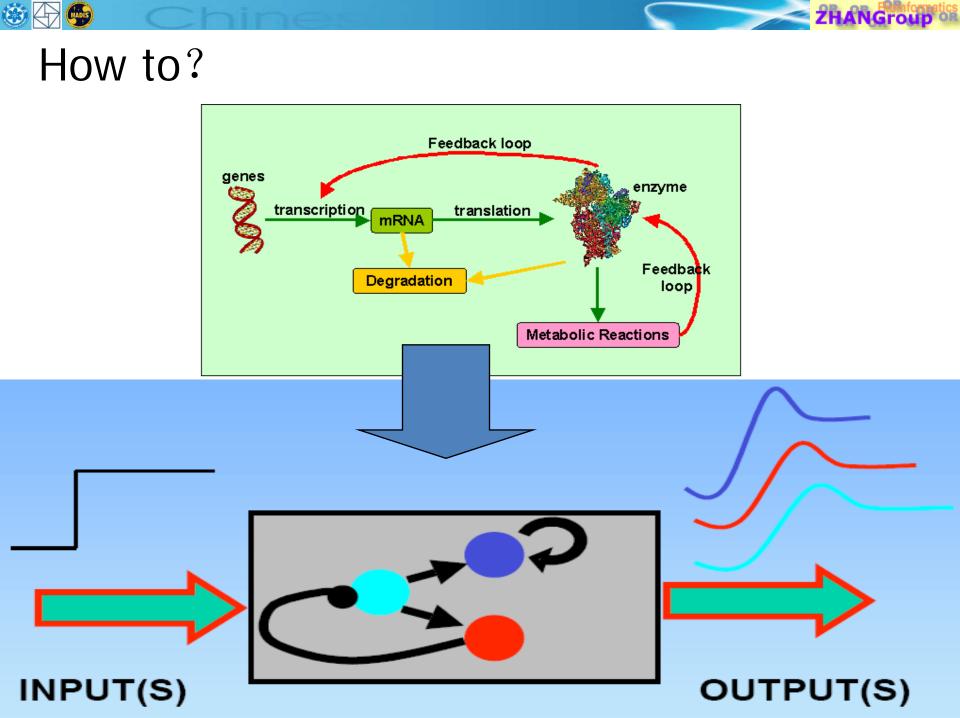




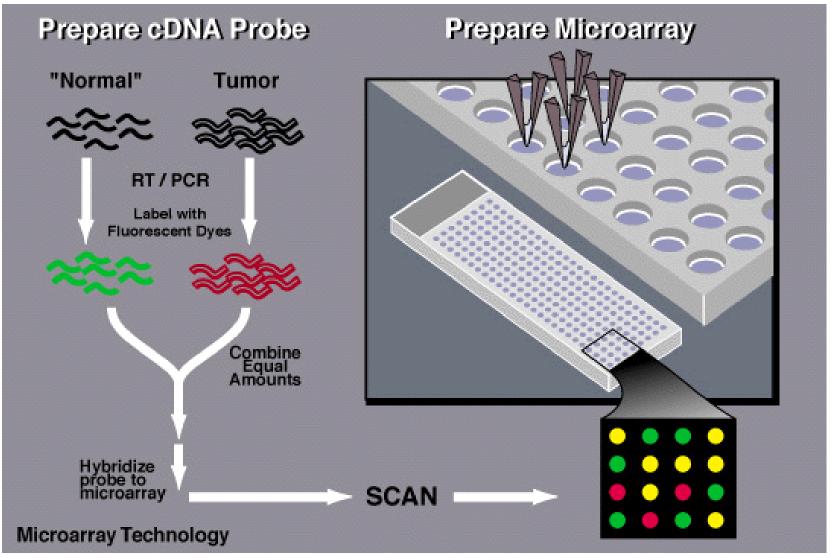


#### Bioremediation

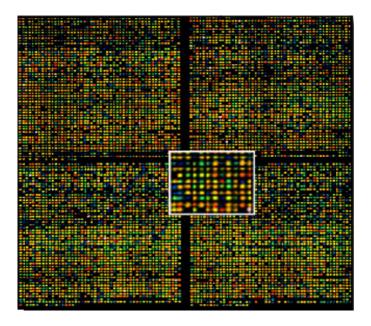










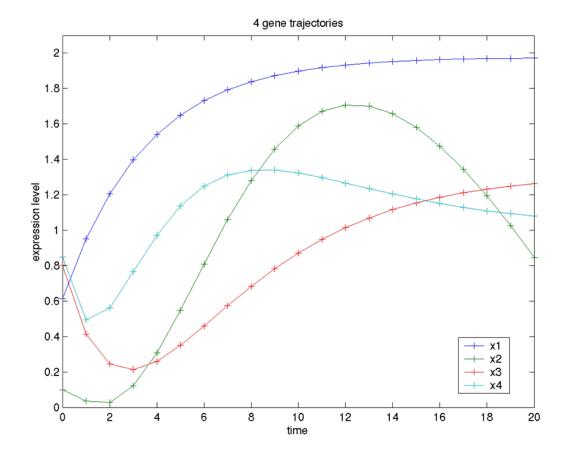


- Experiment design
- Noise reduction
- Normalization
- ...
- Data analysis

Time series (e.g. cell cycle)
Single time point (e.g. steady state)

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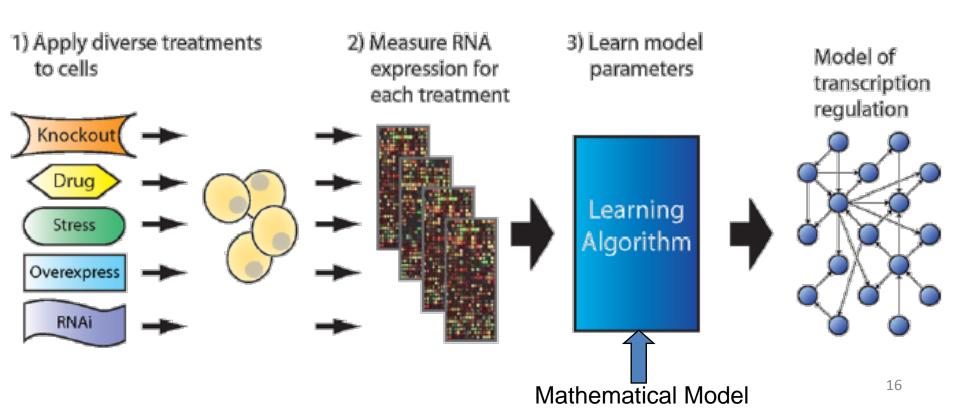






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Goal: Infer structure and function of GRN from expression data

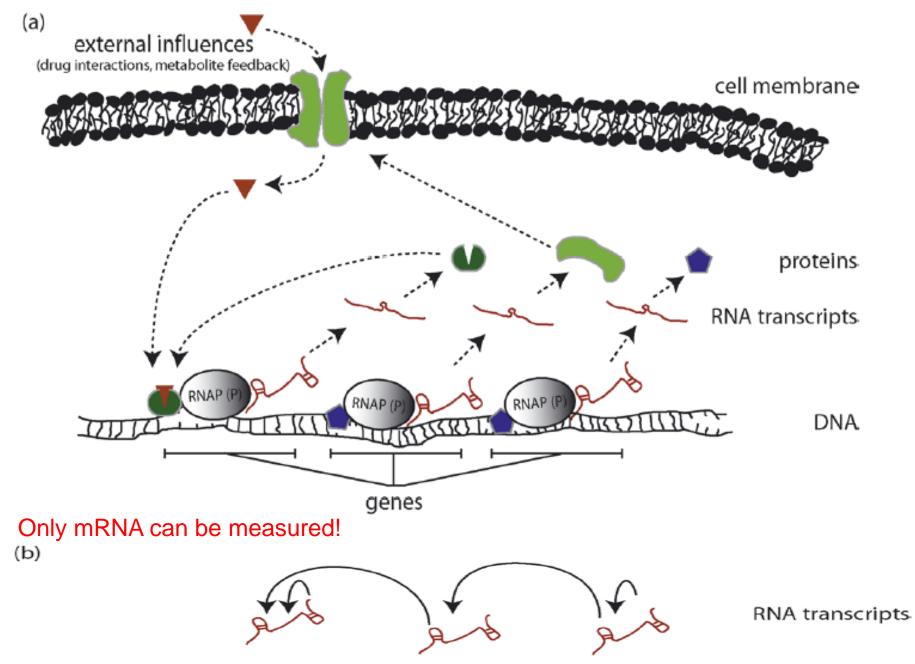


# Outline

- Gene regulatory network modeling
  - Co-expression
  - Boolean networks
  - Bayesian models
  - Differential equations
- Gene regulatory network inference
  - GRNInfer
  - GNTInfer
  - GNMInfer
  - A detailed example



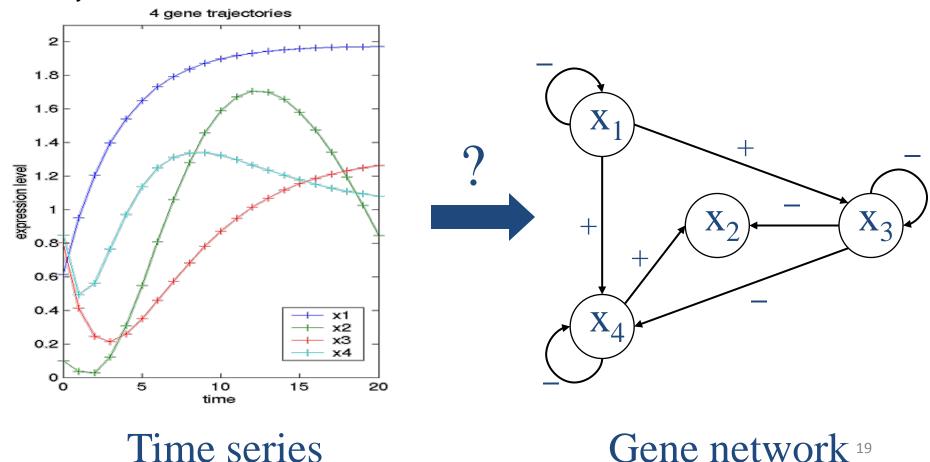




# Gene regulatory network model

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Model can not explicitly represents proteins and metabolites because only RNA can be measured





## Gene Expression Matrix

Given an experiment with *m* genes and *n* assays we produce a matrix *X* where:

 $x_{ij}$  = expression level of the *i*<sup>th</sup> gene in the *j*<sup>th</sup> assay.

$$\mathbf{X} = \begin{pmatrix} x_{11} & \dots & x_{1j} & \dots & x_{1n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ x_{i1} & \dots & x_{ij} & \dots & x_{in} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ x_{m1} & \dots & x_{mj} & \dots & x_{mn} \end{pmatrix}$$

 $g_i$  = Transcriptional response of the *i*<sup>th</sup> gene

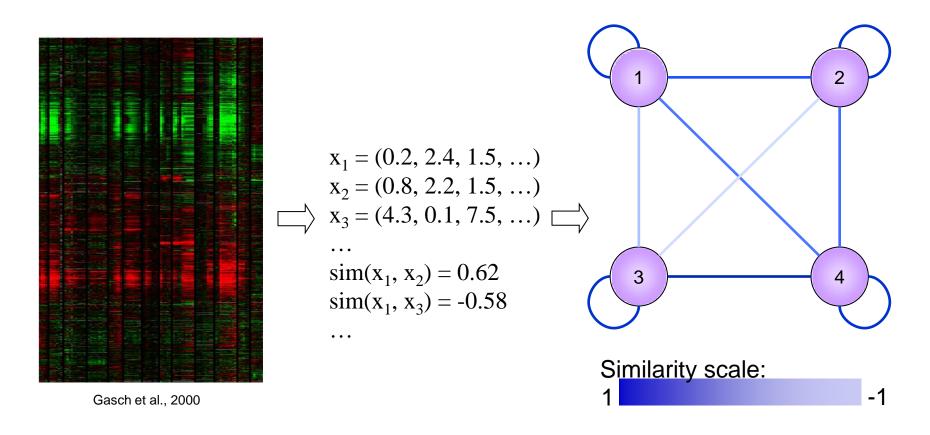
 $a_j$  = Expression profile of the  $j^{\text{th}}$  assay

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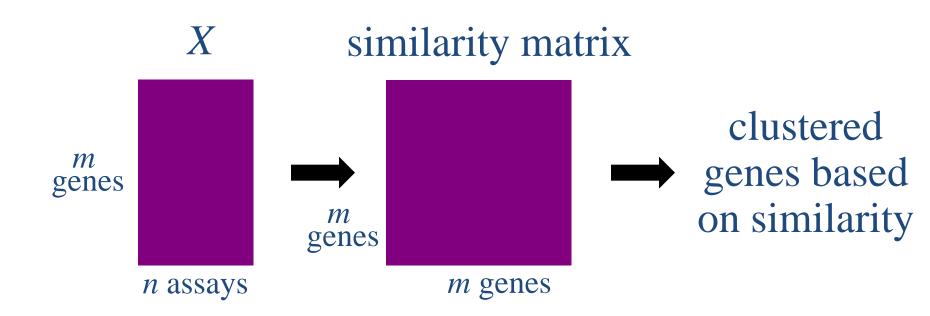
• Gene expression

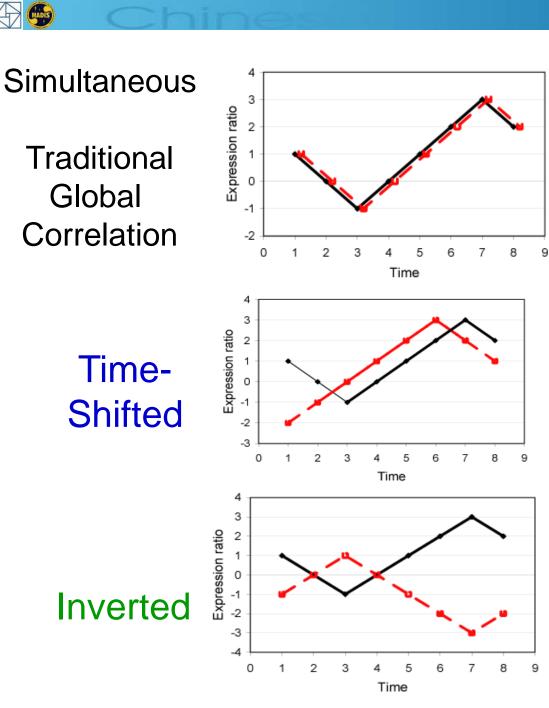




- Clustering genes:
  - Classify genes by their transcriptional response and get an idea of how groups of genes are regulated.
  - Potentially infer functions of unknown genes.
  - Construct relevance network (Co-regulation)
- Clustering assays:
  - Classify diseased versus normal samples by their expression profile.
  - Track the expression levels at different stages in the cell.
  - Study the impact of external stimuli.



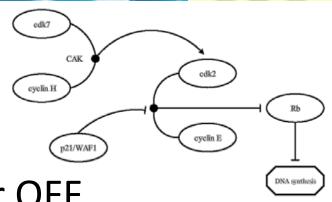




Local Clustering algorithm identifies further (reasonable) types of expression relationships

(Algorithm adapted from **local** sequence alignment)

# Boolean Networks

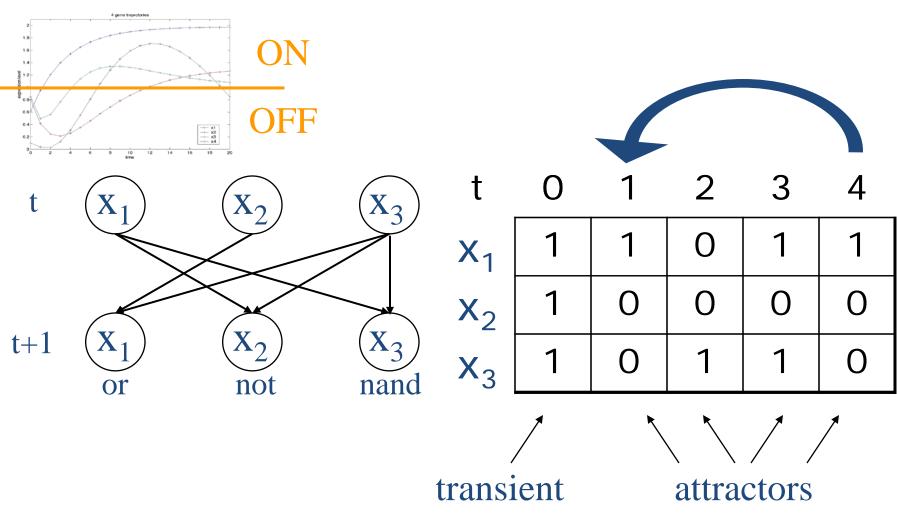


- Genes are assumed to be ON or OFF.
- At any given time, combining the gene states gives a *gene activity pattern* (GAP).
- Given a GAP at time *t*, a deterministic function (a set of logical rules) provides the GAP at time *t* +1.
- GAPs can be classified into *attractor* and *transient* states.



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## Issues with Boolean Networks

- Gene trajectories are continuous and modeling them as ON/OFF might be inadequate.
- A deterministic set of logical rules forces a very stringent model.
  - It doesn't allow for external input.
  - Very susceptible to noise.
- Probability Boolean Networks aims at fixing some of these issues by combining multiple sets of rules.



# **Bayesian Networks**

- A gene regulatory network is represented by directed acyclic graph:
  - Vertices correspond to genes.
  - Edges correspond to direct influence or interaction.
- For each gene x<sub>i</sub>, a conditional distribution
   p(x<sub>i</sub> | ancestors(x<sub>i</sub>)) is defined.
- The graph and the conditional distributions, uniquely specify the joint probability distribution.

### **Bayesian Network Example**

 $\mathbf{X}_{1}$   $\mathbf{X}_{2}$  $\mathbf{X}_{4}$   $\mathbf{X}_{3}$  $\mathbf{X}_{5}$ 

Conditional distributions:  $p(x_1), p(x_2), p(x_3 | x_2),$  $p(x_4 | x_1, x_2), p(x_5 | x_4)$ 

 $p(X) = p(x_1) p(x_2 | x_1) p(x_3 | x_{1,x_2}) p(x_4 | x_{1,x_2,x_3}) p(x_5 | x_{1,x_2,x_3,x_4})$  $p(X) = p(x_1) p(x_2) p(x_3 | x_2) p(x_4 | x_{1,x_2}) p(x_5 | x_4)$ 

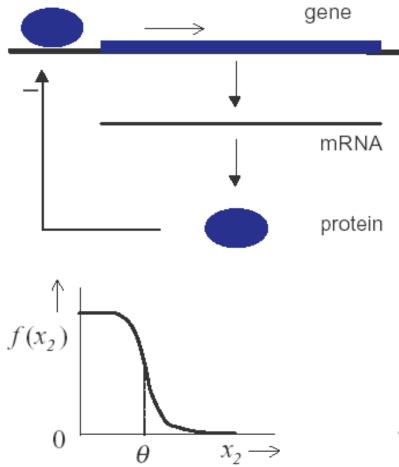
# Learning Bayesian Models

- Using gene expression data, the goal is to find the Bayesian network that best matches the data.
- Recovering optimal conditional probability distributions when the graph is known is "easy".
- Recovering the structure of the graph is NP-hard.

# **Issues with Bayesian Models**

- Computationally intensive.
- Requires lots of data.
- Does not allow for feedback loops which play an important role (Network Motifs).
- Does not make use of the temporal aspect of the data.
- Dynamical Bayesian Networks aim at solving some of these issues but they require even more data.

## **Differential Equation Model**



 $x_1 = mRNA$  concentration  $x_2 = protein$  concentration

$$\dot{x}_1 = \kappa_1 f(x_2) - \gamma_1 x_1$$
$$\dot{x}_2 = \kappa_2 x_1 - \gamma_2 x_2$$

 $\kappa_1$ ,  $\kappa_2 > 0$ , production rate constants  $\gamma_1$ ,  $\gamma_2 > 0$ , degradation rate constants

$$f(x_2) = \frac{\theta^n}{\theta^n + x_2^n}$$
,  $\theta > 0$  threshold

# Linearization

- Typically uses linear differential equations to model the gene trajectories: dx<sub>i</sub>(t) / dt = a<sub>0</sub> + a<sub>i,1</sub>x<sub>1</sub>(t) + a<sub>i,2</sub>x<sub>2</sub>(t) + ... + a<sub>i,n</sub>x<sub>n</sub>(t) + u(t)
- Reasons for that choice:
  - lower number of parameters implies that we are less likely to over fit the data
  - sufficient to model complex interactions between the genes

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### Issues with Differential Equations

- Even under the simplest linear model, there are *m(m+1)* unknown parameters to estimate:
  - *m(m-1) directional* effects
  - *m self* effects
  - *m* constant effects
- Number of data points is *m* and we typically have that *n* << *m* (few time-points).
- Extra constraints must be incorporated into the model such as:
  - Sparse structure of the network
  - Other prior information

## References

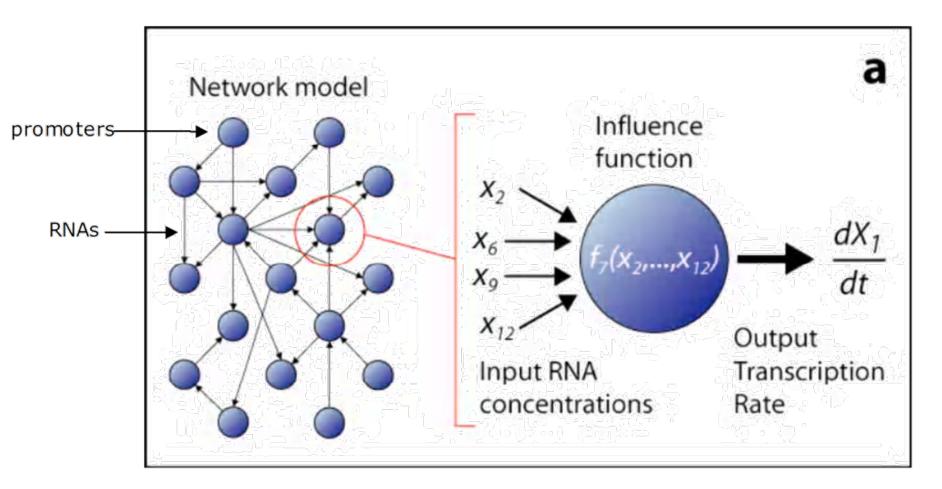
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  - Lian et al. (1998), PSB, 3: 18-29.
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  - Friedman et al. (2000), *RECOMB 2000*.
  - Hartemink et al. (2001), PSB, 6: 422-433.
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  - D'haeseleer et al. (1999), *PSB*, 4: 41-52.
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# Outline

- Gene network modeling
  - Co-expression
  - Boolean networks
  - Bayesian models
  - Differential equations
- Gene regulatory network inference
  - GRNInfer
  - GNTInfer
  - GNMInfer

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## ODE model

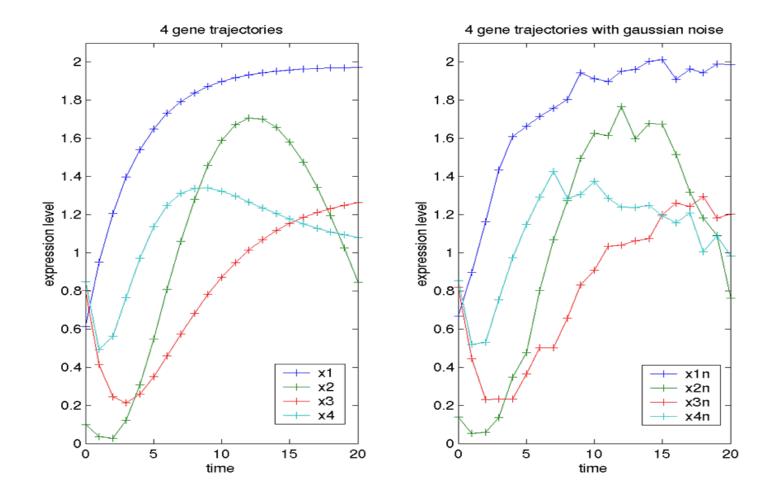


$$dX_{1}/dt = f_{7}(X_{2},...) = a_{2} X_{2} + a_{6} X_{6} + a_{9} X_{9} + a_{12} X_{12}$$



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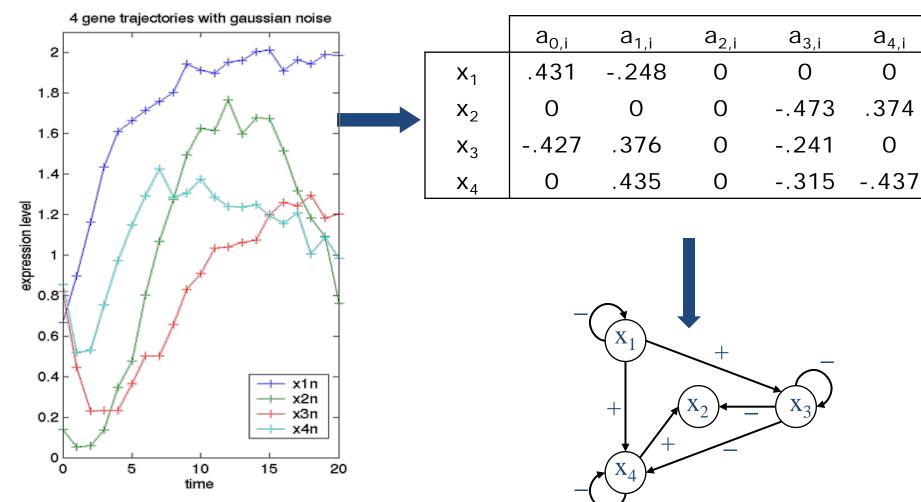
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We add gaussian noise to model errors.



## Network Inference

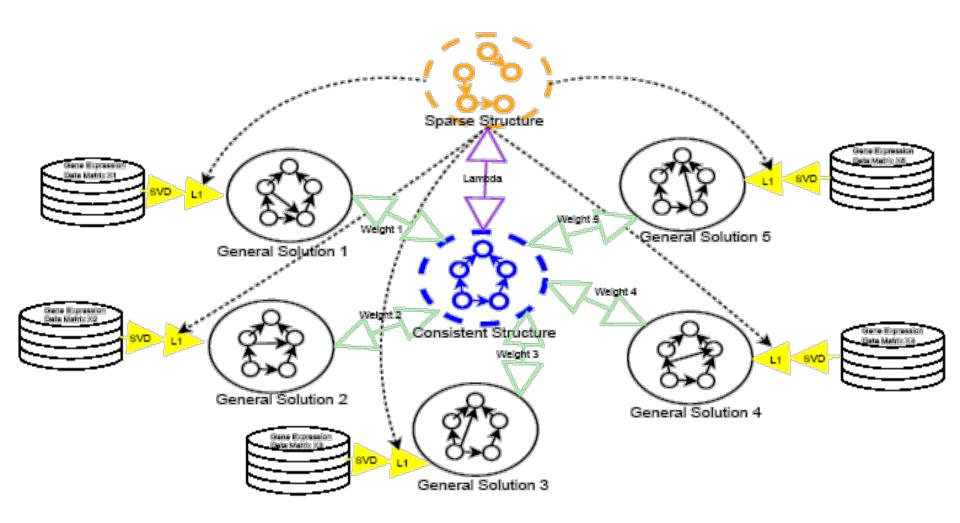


## GRNInfer (Gene Regulatory Network reconstruction tool)

- A single dataset consists of relatively few time points (less than 20) but a large number of genes (in thousands)
- Multiple Gene expression datasets are generated by different groups worldwide are increasingly accumulated on many species
- Combining and further exploiting multiple datasets in an integrative and systematic manner, the scarcity of data can be greatly alleviated.
- A more accurate reconstruction of GN can be expected.
- Simply arranging multiple time-course datasets into a single timecourse dataset is Inappropriate for GN inference due to data normalization issues and lack of temporal relationships among datasets.
- A biological gene network is expected to be sparse



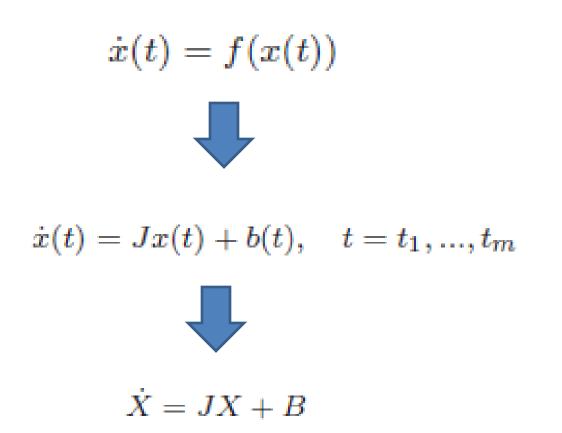
## **GRNInfer scheme**







## General solution of a single dataset



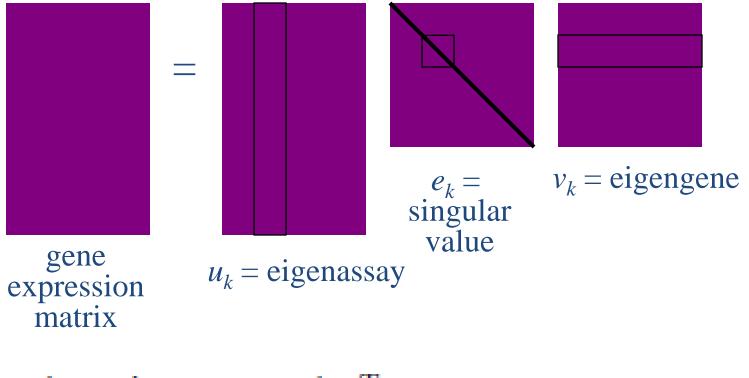


- Curse of dimension: #of experiments <<#of variables m(20)<<n(6000)</li>
- →Inference problem is undetermined
- How to recover J? (Infinitely many possible solutions → many network architecture fit the data)
- Find one possible solution as a particular solution (SVD Singular Value Decomposition)  $J_{n \times n} X_{n \times m} = \dot{X}_{n \times m} - B_{n \times m}$

## Singular Value Decomposition

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$$X^{T}_{m \ge n} = U_{m \ge n} E_{n \ge n} V^{T}_{n \ge n} (m << n)$$



 $\hat{J} = (\dot{X} - B)UE^{-1}V^T$ 



SVD solution is the particular solution in the least square meaning

$$\widehat{J} = argmin \|JX + B - \dot{X}\|_2$$

• General solution: affine space

- Y denotes all degrees of the freedom can be used to optimize some extra criterion
- For example the sparsity of J → Maximize the number of zeros in J

• Impose J=0 
$$\rightarrow$$
 i. e.  $\widehat{J} = -YV^T$ 

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The general solution represents all of the possible networks that are consistent with the single microarray dataset, depending on arbitrary Y.

We will find the most consistent network structure  $J = (J_{ij})_{n \times n}$  for all k = 1, ..., N, with consideration of sparse structure

## **Optimization model**

$$\min_{Y,J} \quad \sum_{k=1}^{N} \sum_{i=1}^{n} \sum_{j=1}^{n} [\omega^k |J_{ij} - J_{ij}^k| + \lambda |J_{ij}|]$$

## **Decomposition Algorithm**

**STEP-0**: Initialization. Obtain all of the particular solution  $\hat{J}^k$  by SVD, and  $\omega^k$ . Set initial value  $J_{ij}(0) = 0$ ,  $Y_{ij}^k(0) = 0$  and  $J_{ij}^k(0) = \hat{J}^k$ , and positive  $\lambda$ ,  $\epsilon$ . Set q = 1.

**STEP-1**: Set  $J^k(q) = J^k(q-1) + Y^k(q)V_k^T$ and solve  $y_{ij}^k(q)$  at iteration q by LP with J(q-1) fixed, i.e. solve  $Y^k(q) = (y_{ij}^k(q))_{m \times m}$  of the following subproblem for k = 1, ..., N with J(q-1) given  $(y_{ij}^k(q) = 0$  if  $j > l_k)$ 

$$\min_{Y^k(q)} \sum_{i=1}^n \sum_{j=1}^n |J_{ij}(q-1) - J_{ij}^k(q)|$$

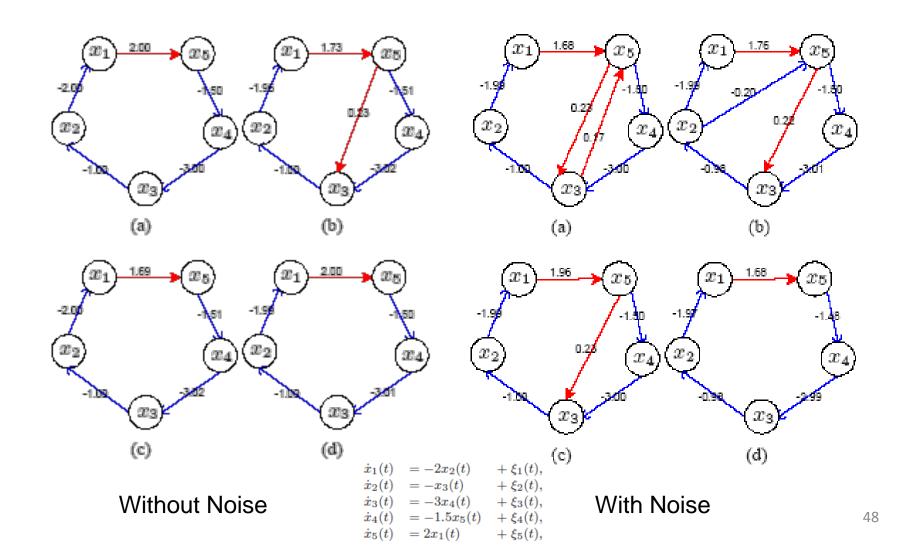
**STEP-2**: Solving  $J_{ij}(q)$  at iteration q by LP with all of  $y_{ij}^k(q)$  given, i.e. solve J(q) of the following problem with all of  $J^k(q)$  fixed.

$$\min_{J(q)} \sum_{k=1}^{N} \sum_{i=1}^{n} \sum_{j=1}^{n} [\omega^{k} |J_{ij}(q) - J_{ij}^{k}(q)| + \lambda |J_{ij}(q)|]$$

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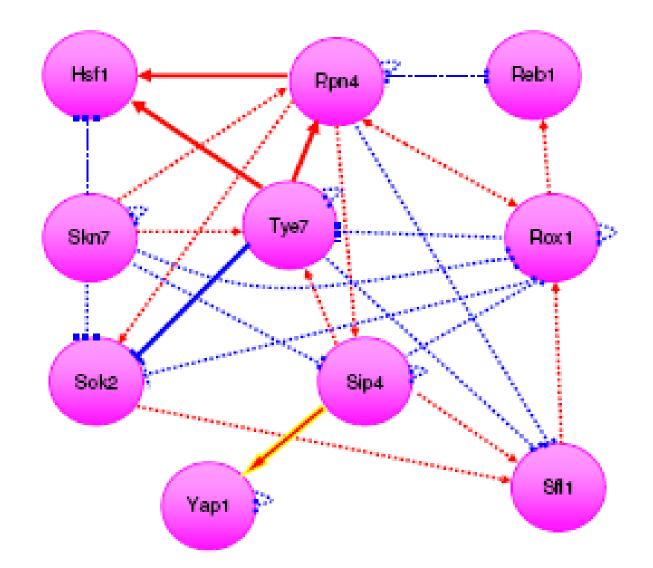


## Simulated examples



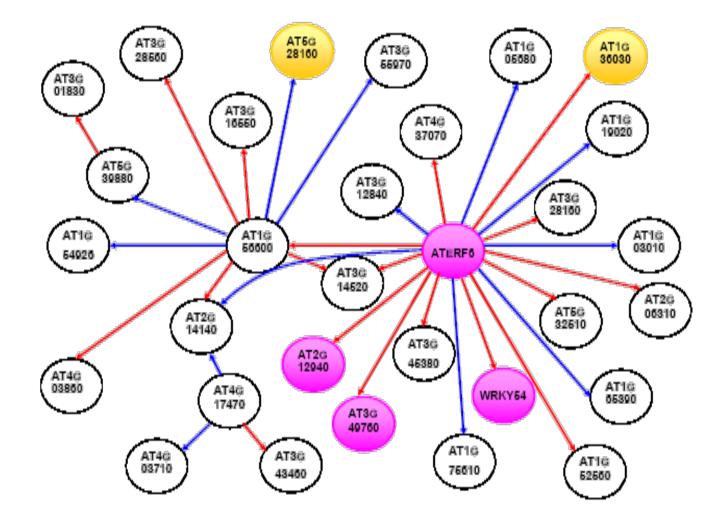


## Heat-Shock Response for Yeast



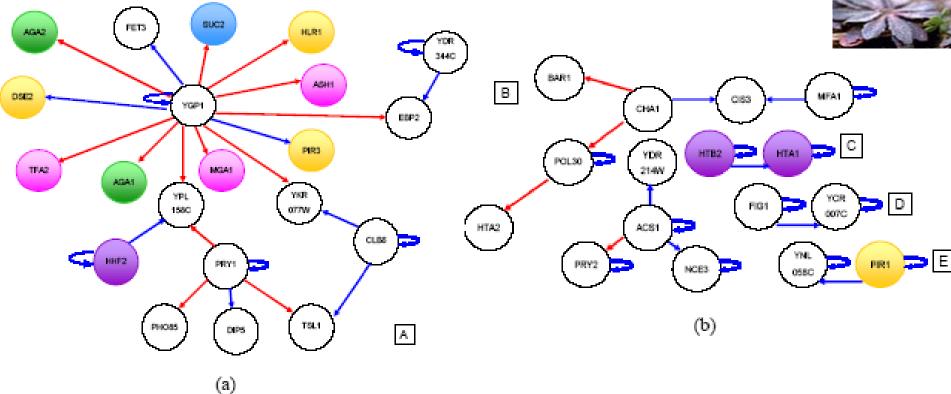


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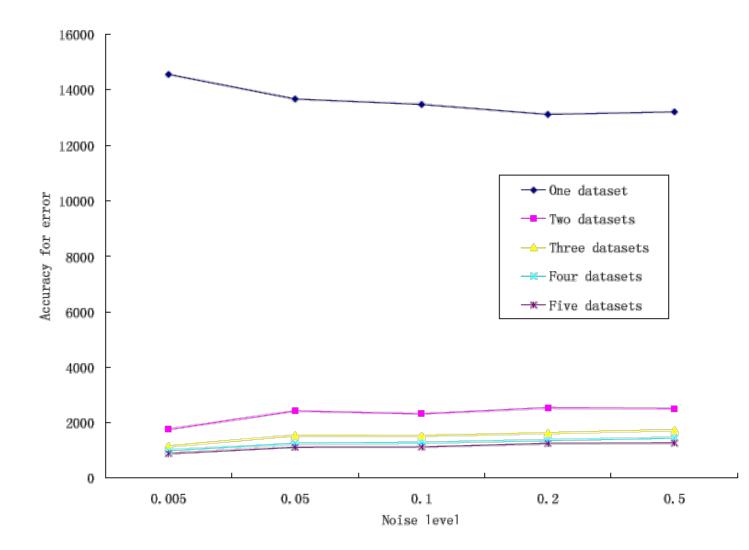


# Arabidopsis (9 Datasets)



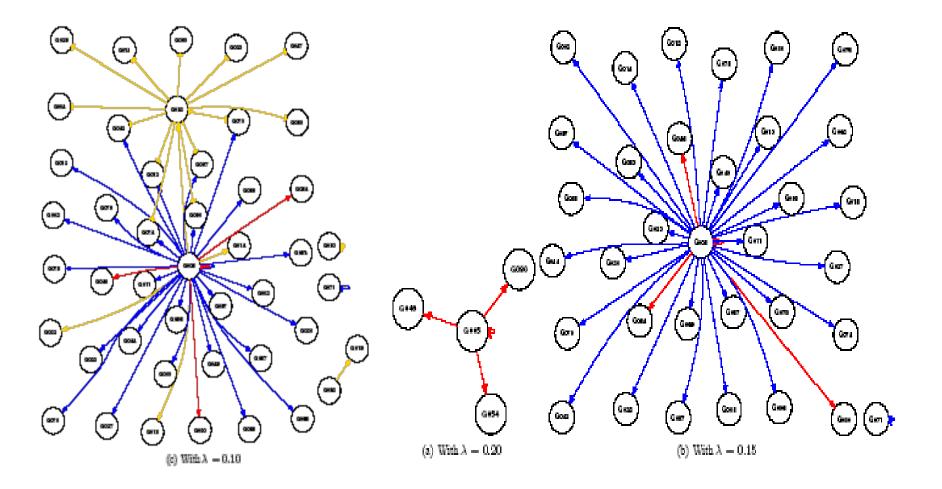


## The advantage of multiple datasets



## Consistent structure

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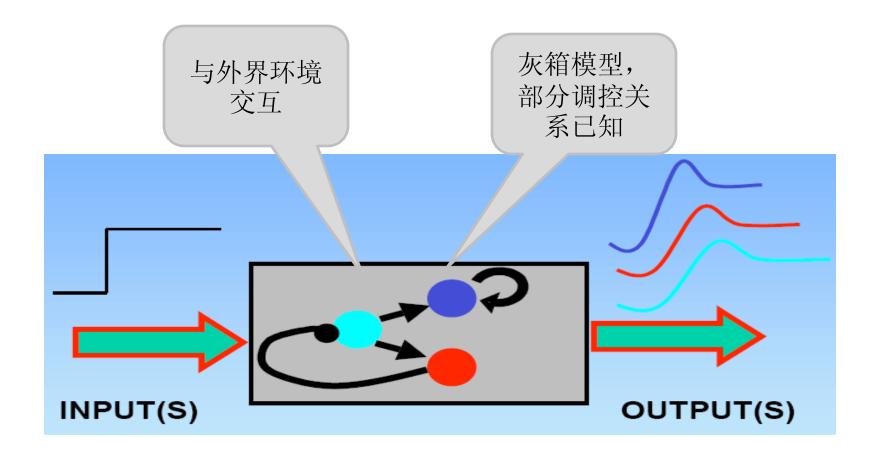




- Include other available information derived from expression profile and from published literature so as to recover gene regulations in a more robust and reliable manner.
- Incorporate external inputs or perturbations into the formulation so that molecular targets (genes) can be identified in a systematic way.











考虑外部输入或者扰动对基因调控网络的影响,用系统的方式识别他们的靶点基因

- 可以考虑的外部因素:
- •环境因素:温度、压力
- 药物或化合物
- 非编码RNA
- 基因敲除
- 其它



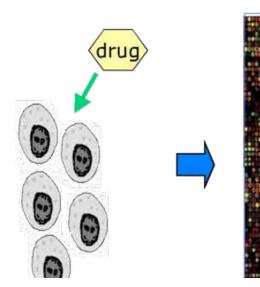
# Identify compound Targets

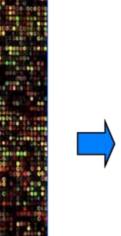
Treat cells with drug compound

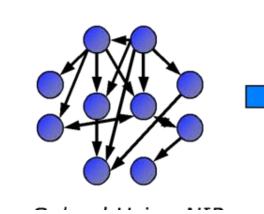
Obtain expression profile

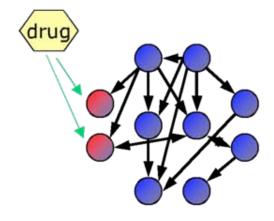
Filter profile using identified network

Identify genetic mediators of drug activity









数学表达

• 引入控制项

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•  

$$X = J_{n \times n} X + P_{n \times s} C + \varepsilon$$

$$X(1), \dots, X(m), C(1), \dots, C(t) \Longrightarrow J_{n \times n}, P_{n \times s}$$

$$X(t) \in \mathbb{R}^{n}, C(t) \in \mathbb{R}^{s} \qquad m \ll n$$

• P代表s个外部扰动对各个基因的影响

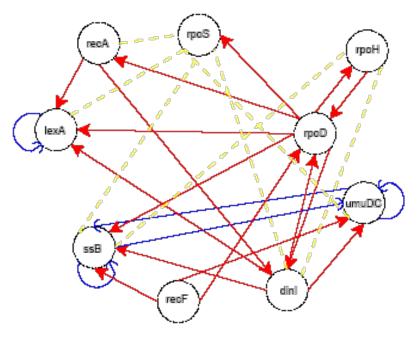
• 已知X, C, 求矩阵 • 
$$X = [J, P] \begin{bmatrix} X \\ C \end{bmatrix}$$



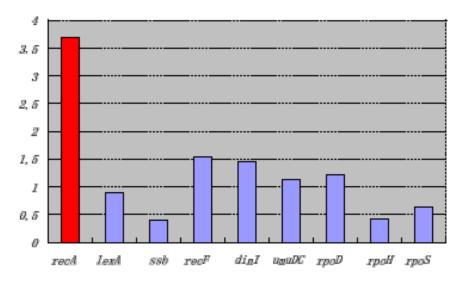


## E. Coli SOS Pathway

	recA	lexA	$\operatorname{ssb}$	$\mathrm{recF}$	dinI	umuDC	rpoD	rpoH	$_{\rm rpoS}$	Perturbation
recA	-0.0682	0.1149	0.0599	-0.0095	-0.0431	0.0000	0.0173	-0.0104	0.0000	0.1739
lexA	0.0009	-0.1098	0.0232	-0.0197	0.0061	0.0000	0.0082	0.0384	0.0000	0.0418
ssb	-0.0181	0.0188	-0.0141	0.0279	0.0020	-0.0192	0.0018	0.0000	0.0000	0.0187
recF	-0.0424	0.0015	0.0539	-0.0863	0.0000	-0.0090	-0.0005	0.0398	0.0000	0.0731
dinI	0.0268	0.0239	0.0538	0.0000	-0.0827	0.0769	0.0177	0.0000	0.0000	0.0689
umuDC	0.0000	0.0000	-0.0527	0.0247	0.0280	-0.0705	0.0000	0.0083	0.0000	0.0531
rpoD	-0.0525	0.0237	0.0145	0.0009	0.0059	0.0000	-0.0211	0.0336	0.0000	0.0578
rpoH	-0.0256	-0.0143	0.0000	-0.0111	0.0000	0.0335	0.0127	-0.0032	0.0000	0.0195
rpoS	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0101	0.0091	-0.0274	0.0304



(a) Predicted network structure



(b) Predicted perturbation





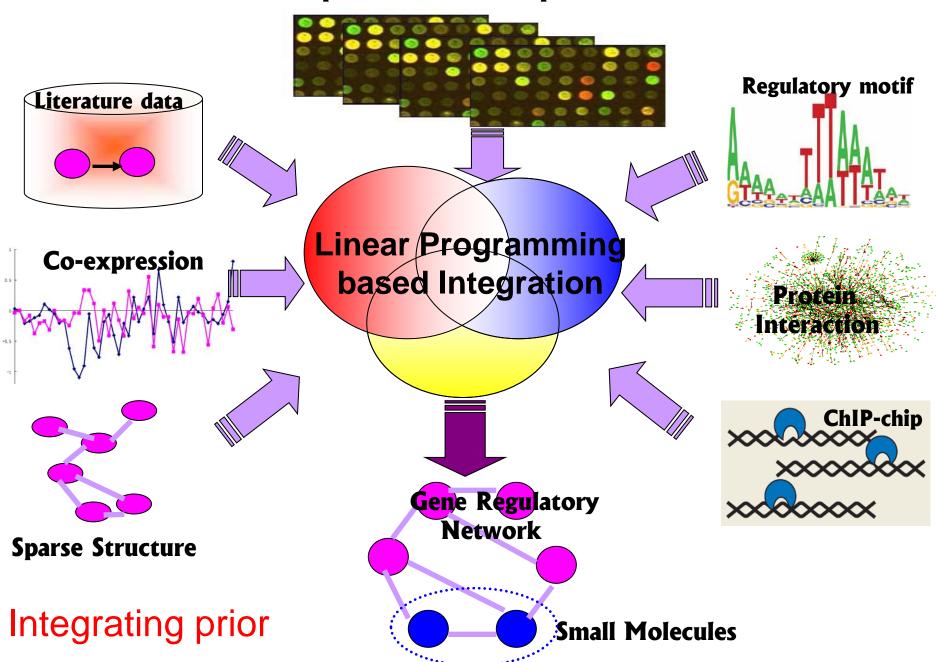
1. 基因调控网络中的维度问题

2. 大量的关于基因调控网络的异源数据

3. 集成大量的先验信息有助于缓解数据稀缺状况

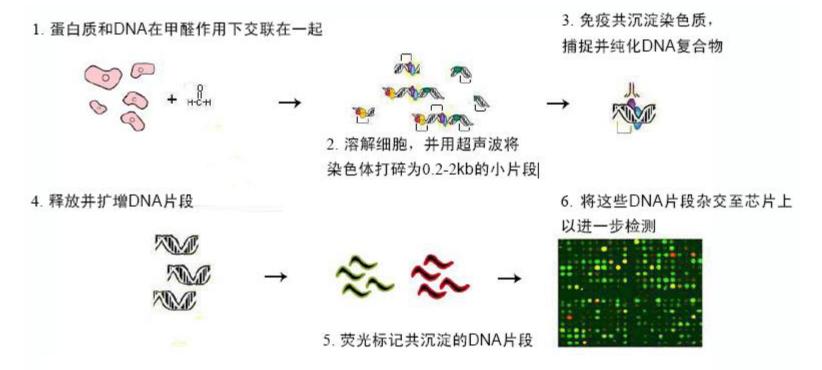
4. 同时使得得到的调控网络更加精确。

**Multiple Time-course Expression Data** 





# 染色体免疫共沉淀技术(Chromatin Immunoprecipitation, ChIP)



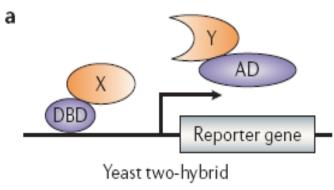
染色体免疫共沉淀在过去十年已经成为表观遗传信息研究的主要方法,确定转录因子及其结合位点

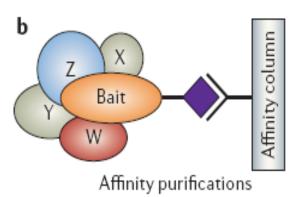


## 实验方法预测蛋白相互作用

#### Box 1 | Uncovering protein interactions

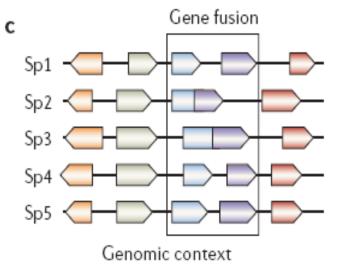
**Experimental methods** 

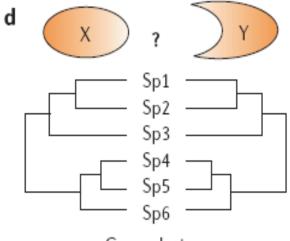




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#### **Computational methods**

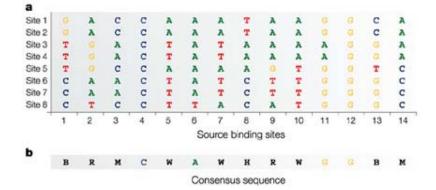




Co-evolution



Chine



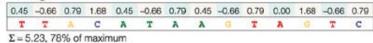
#### c Position frequency matrix (PFM)

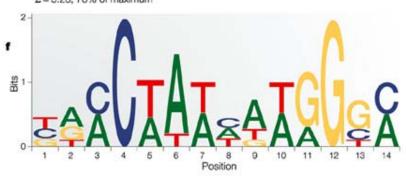
- [	1	2	3	4	5	6	7	8	9	10	11	12	13	14
A	0	4	4	0	3	7	4	3	5	4	2	0	0	4
c	3	0	4	8	0	0	0	3	0	0	0	0	2	4
G	2	3	0	0	0	0	0	0	1	0	6	8	5	0
т	3	1	0	0	5	1	4	2	2	4	0	0	1	0

#### d Position weight matrix (PWM)

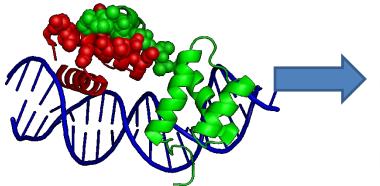
A	-1.93	0.79	0.79	-1.93	0.45	1.50	0.79	0.45	1.07	0.79	0.00	-1.93	-1.93	0.79
C	0.45	-1.93	0.79	1.68	-1.93	-1.93	-1.93	0.45	-1.93	-1.93	-1.93	-1.93	0.00	0.79
G	0.00	0.45	-1.93	-1.93	-1.93	-1.93	-1.93	-1.93	0.66	-1.93	1.30	1.68	1.07	-1.93
т	0.15	0.66	-1.93	-1.93	1.07	0.66	0.79	0.00	0.00	0.79	-1.93	-1.93	-0.66	-1.93

#### e Site scoring





Nature Reviews | Genetics





# 先验信息分类

- 无向(Undirected): 仅仅知道有无调控关系。 例如蛋白质相互作用数据以及共表达数据
- 有向无符号(Directed and un-signed). 知道有 方向的调控关系,但是不知道是激活还是 压制作用。例如ChIP-chip 数据和motif 出现 数据
- 有向有符号(Directed and signed). 知道有方向的调控关系,同时知道是激活还是压制作用,但是没有调控的强度数据,例如文献中记录的调控关系。





有很多对网络结构推断有价值的的先验信息,例如从数据 库或文献中得到的基因间调控数据,这些信息可通过添加 线性规划的约束来提高所得到的聚合网络的精度。

$$\begin{split} \min_{Y^{1},Y^{2},...,Y^{N},L} & \sum_{k=1}^{N} \sum_{i=1}^{n} \sum_{j=1}^{n+s} \omega_{k} \mid L_{ij} - L_{ij}^{k} \mid + \lambda \sum_{(i,j) \in \{(i,j) \mid K_{ij} = 0 \text{ or } U_{ij} = 0\}} \sum_{i,j \in \{1,2,...,n\}} L_{ij} > 0 & \text{if } K_{ij} > 0 & i,j \in \{1,2,...,n\} \\ L_{ij} < 0 & \text{if } K_{ij} < 0 & i,j \in \{1,2,...,n\} \\ L_{ij} = 0 & \text{if } E_{ij} = 0 & i,j \in \{1,2,...,n\} \end{split}$$

硬约束:已知信息较为精确,希望在推断的网络中体现 软约束:噪声较大的信息,在推断的网络中出现与否取决于其他数据的相容性

## GNNInfer(Gene Network reconstruction tool with Modular structure)

- Primary literature and information in databases for well-studied organisms such as E. coli and S. cerevisiae indicated the complex network takes network motifs and modules as its basic building block.
- Introducing the assumption is a cellular system is composed of locally interacting biological modules.
- Integrate the bottom-up and top-down reconstruction strategies.
- Initially perform a network modules identification. Then the modular gene regulatory network inferred from multiple microarray datasets To relieve the curse of dimension.
- To ensure sparse network in a structured way.



# Top-down methodology

- Inferring a regulatory network without a priori knowledge
- TOP-DOWN APPROACH: the architecture of the network is inferred (or reverse engineered) based on the observed response of the system to a series of experimental perturbations.
- In engineering sciences: system identification

1. typical use: large scale modeling from high throughput data (genomic/proteomic/metabolomic)

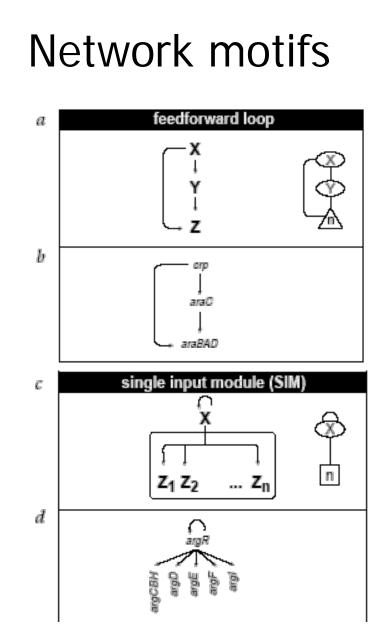
2. main use: gene networks, any kind of complex network (metabolic, signalling pathways, protein activity, etc.)

# Bottom-up methodology

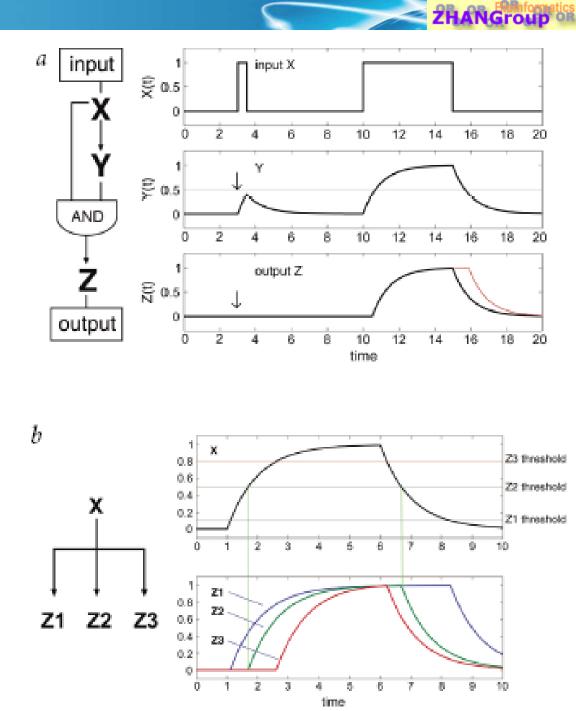
- Mathematical model was obtained from already available knowledge of the mechanisms of action/interaction between to or more components
- **BOTTOM-UP APPROACH**: model built from a priori biological information
- Advantages:
  - 1. readily testable comparing simulation vs experiments
  - 2. allows to model known pathways
  - 3. allows to pass from qualitative to quantitative analysis

### Drawbacks

- 1. can model only known molecular processes
- 2. does not allow to discover new pathways
- 3. less applicable to poorly characterized networks
- 4. useful mainly for small/medium scale systems



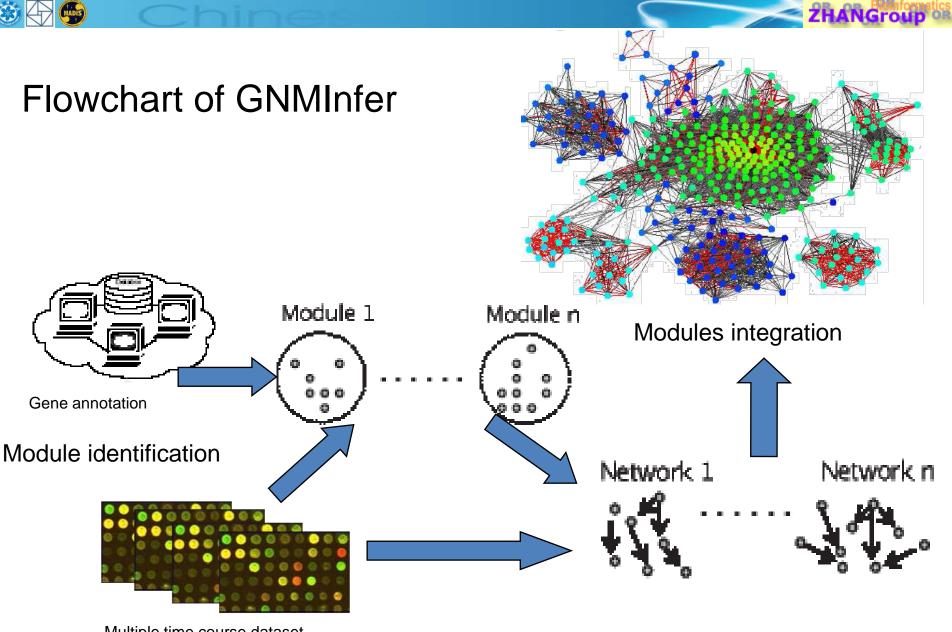
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## Network Modules

- Topological module: most of the genes are likely to be related to the genes in the same module rather than the genes in different modules. (Clustering on the expression data to find the co-regulations relationships)
- Functional module: most of the genes are likely to have similar function related to the genes in the same module rather than the genes in different modules. (Clustering the gene annotation data to find the similar function relationships)

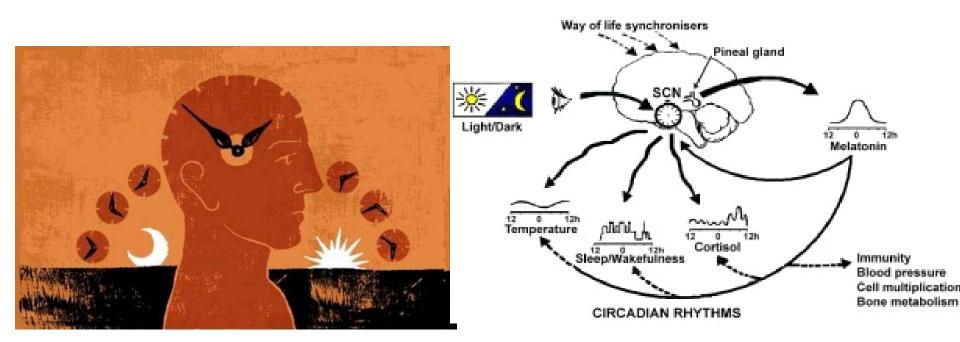


Multiple time course dataset

# An Example: Circadian rhythm

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• Circadian rhythm is fundamentally important in physiological processes of mammals.



# Why gene regulatory network

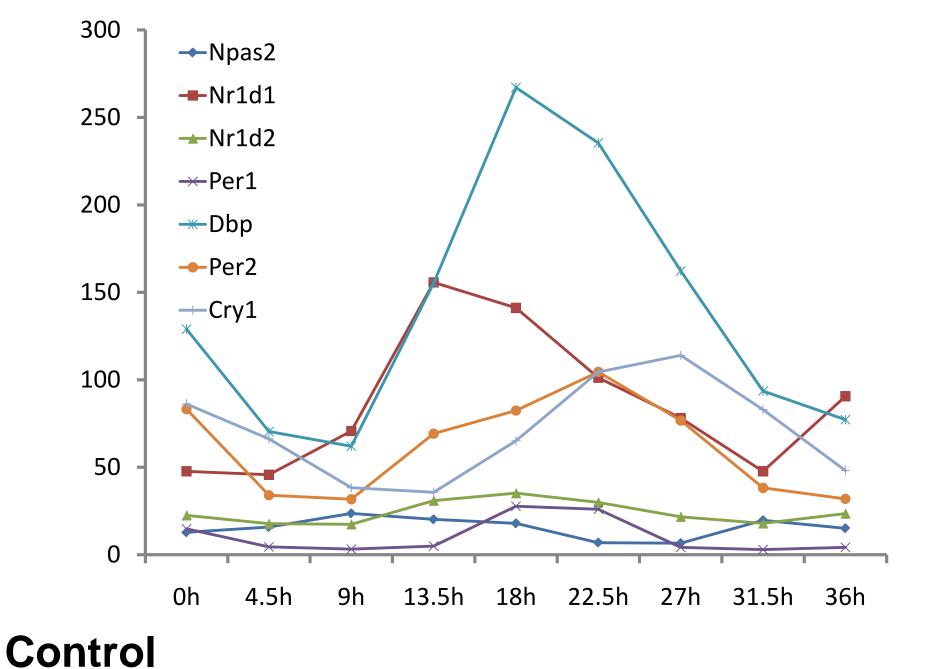
- The 20,000 dissociated neurons consisting of a pair of the mammalian uprachiasmatic nuclei (SCN) display autonomous rhythms in electrophysiological activities. This indicates that the oscillator mechanism resides within individual cells
- Recent observations revealed that a large number of genes undergo circadian oscillation in their expression levels.
- Furthermore, extensive studies have identified that a set of key circadian genes utilize the transcriptional-translational auto-regulatory loop to generate molecular oscillations of the "central clock".



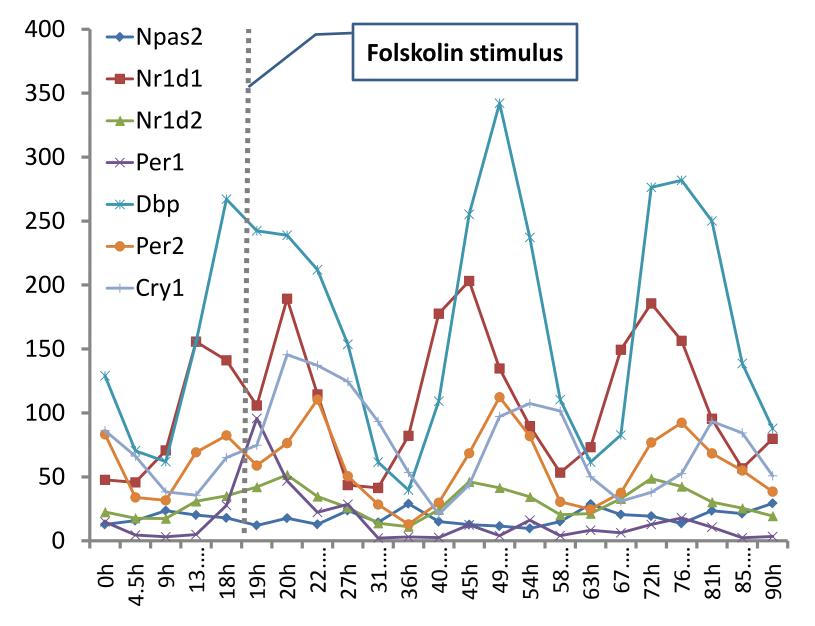
- The laboratory cultured cells from SCN
- Perturbation: Forskolin stimuli can reset the clock of the cells by phase advance and phase delay.
- Four time-series microarray
- 1. Control, 0-36 hour, **14** time points;
- 2. CT6, 0-90 hour, drug is applied at 18 hour, **16** time points;
- 3. CT14, 0-90 hour, drug is applied at 27 hour, **14** time points;
- 4. CT22,0-90 hour, drug is applied at 32 hour, **12** time points.





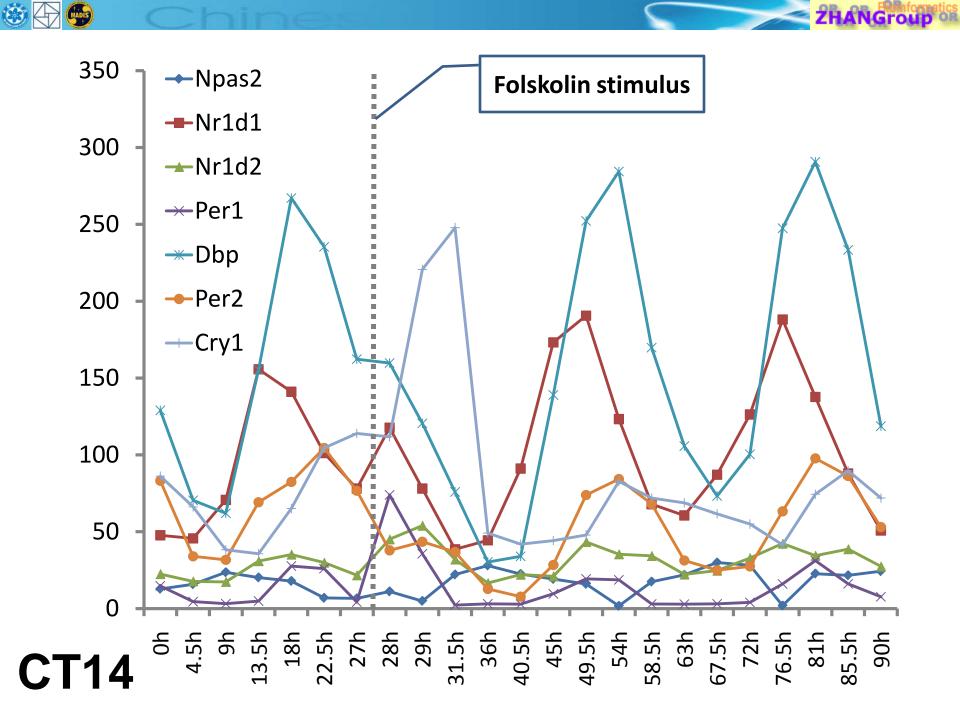


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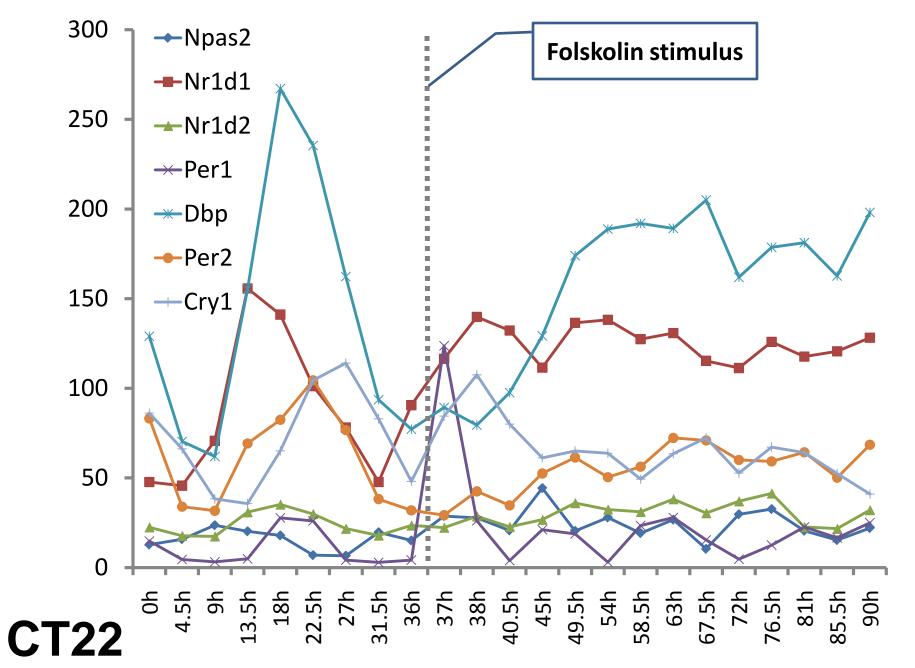


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CT6







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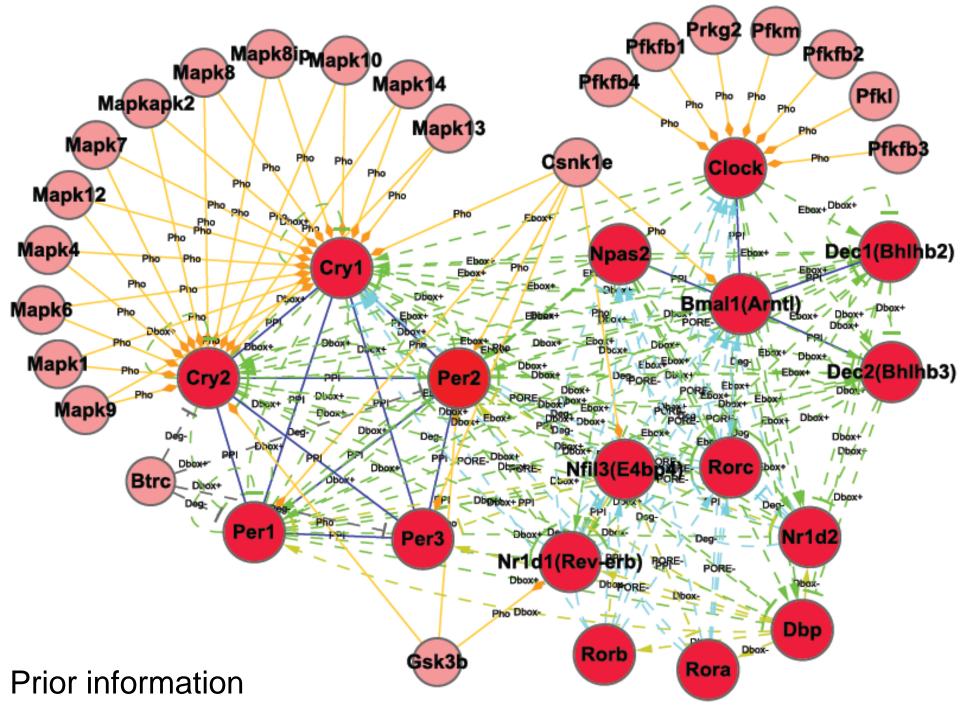
- Key circadian genes: 18 well-studied clock genes
- **Circadian-related genes: 22** genes having protein interactions and phosphorylations relationships with the 18 key circadian genes.
- Oscillatory gene list: 55 genes are identified to see whether typical oscillations exist or not in gene expression data.

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# Prior information

- 14 physical protein interactions
- 40 phosphorylation interactions
- Cis-regulatory element: 134 transcriptional regulatory interactions by linking the transcription factor with their target promoter region in the gene level
- Protein-drug interaction: the significantly induced and or repressed genes are identified as the potential target of the drug folskolin.





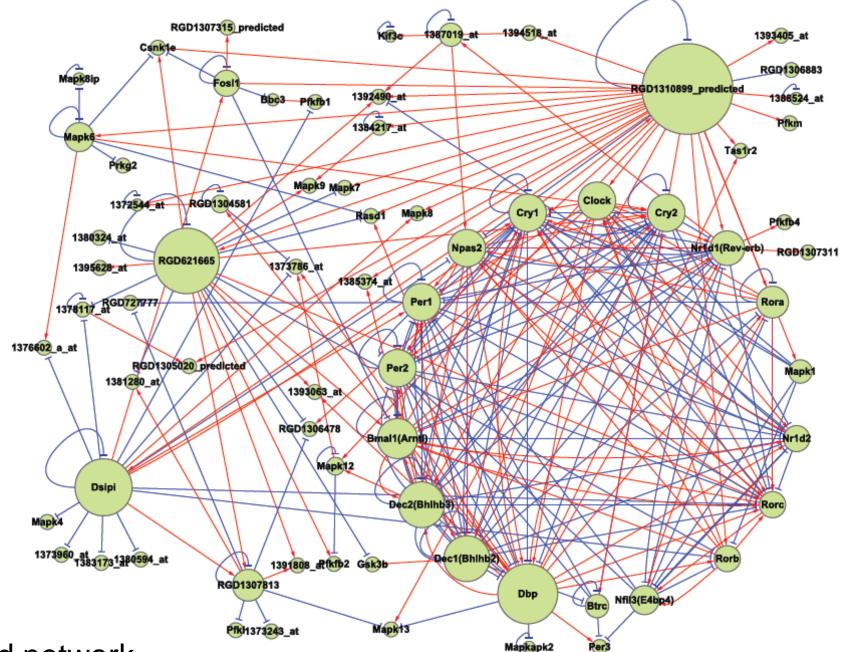


#### Network inference

- 276 predicted regulatory relationships among 80 circadian related genes.
- 138 new regulatory relationships that are not in the prior information (73 activations and 65 repressions)

(a) brand new regulatory relationships(b) signs and weights for those functional relationships in the prior information.

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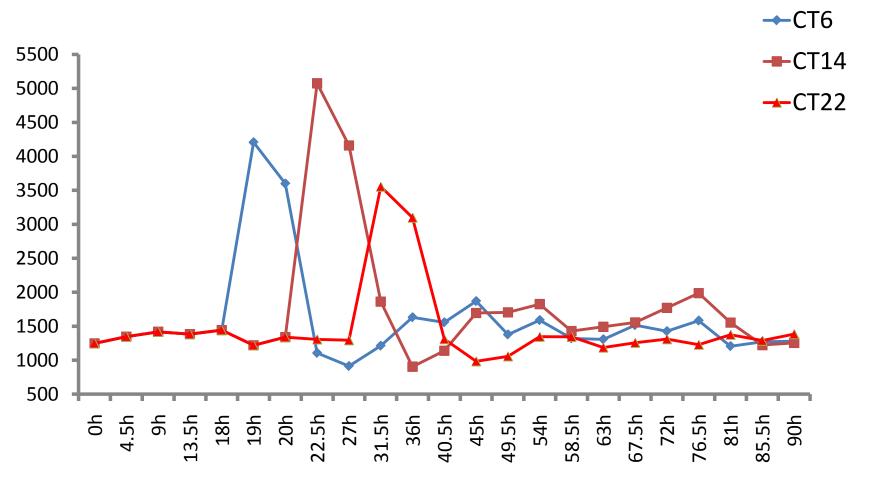
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Inferred network



### Four important hubs

- Dsipi (regulate 17 target genes): A transcription factor protecting T-cells from IL2 deprivationinduced
- *RGD621665* (regulate 20 target genes): a regulator of G-protein signaling
- *RGD1307813* (regulate 8 target genes): related to endoplasmic reticulum,cell redox homeostasis, and protein folding.
- *RGD1310899\_predicted* (regulate 29 target genes)



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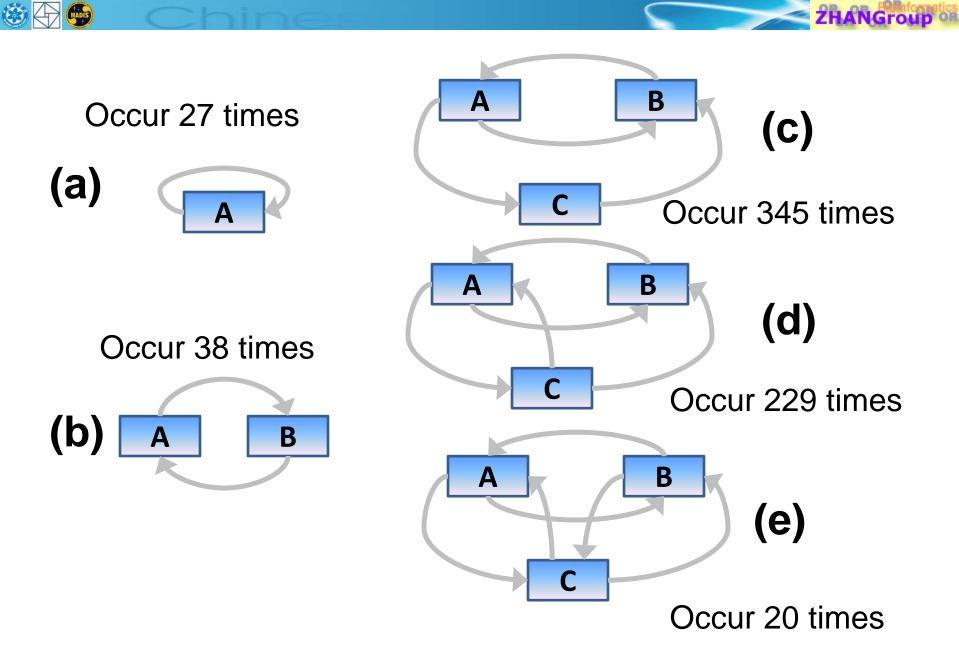
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mRNA expression profile of hub gene RGD621665 (Rgs2)





- Transcription-translation feedback loops are important in driving circadian rhythm. For example, *Bmal1* and *Clock* proteins form a complex that positively regulates the transcription of *Per* and *Cry* family genes.
- z-score and p-value are used to assess the statistical significance of the certain motif in our predicted network against 1000 randomized networks



Enriched feedback motifs (p-value<1e-10)



- Network study enables a system-wide overview on the gene regulation in mechanism of circadian rhythm.
- Our method theoretically ensures the derivation of the most consistent network with all available information.
- Data integration strategy improves the reliability of the inferred gene regulatory network.