Supplementary Material

October 15, 2014

1 Experiment on real networks

To illustrate that the framework can reveal hidden robustness information of community structures in real networks, we apply it to three famous benchmarks comparing with their corresponding randomized networks, i.e. Karate network [1], College football network [1] and Political books network [2]. These results are shown in Table 1.

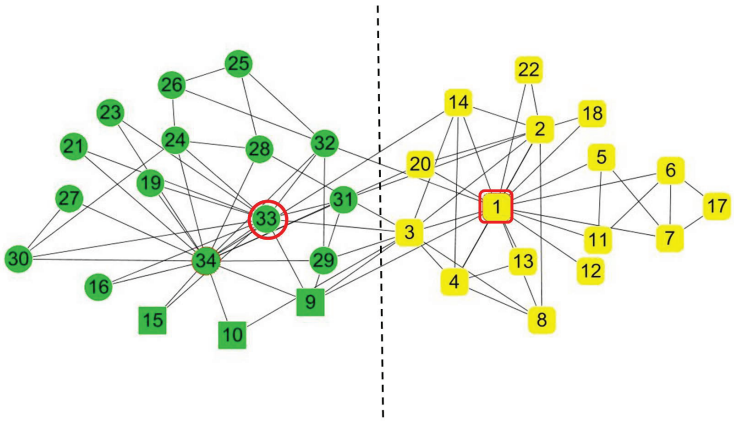
α_C	Real network	Randomized network
Karate network	0.672	0.841
College football network	0.757	0.902
Political books network	0.698	0.881

Table 1: Comparison of α_C between real networks and corresponding randomized networks.

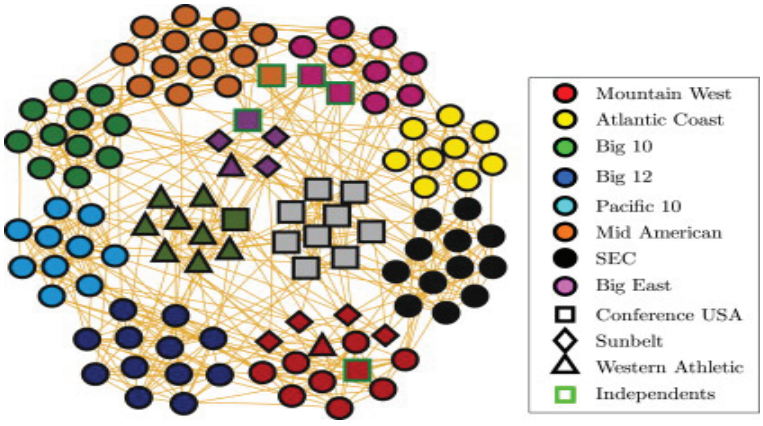
Table 1 shows that the corresponding α_C value of real networks are all smaller than randomized networks. Since random networks own few modularized structures, the results verify the effectiveness of our framework.

2 The relationship between networks establish mechanism and robustness

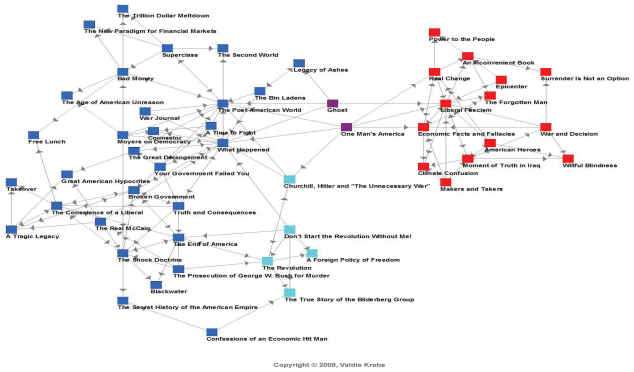
In order to verify our method, it is applied to three famous artificial networks– ER random graph, BA scale-free network, and $P\&S$ network [6] where the number of nodes are 10,000 and the average degree is all 3. The comparison of the clustering coefficients ω and α_C is shown in the following Table 2.



(a)



(b)



(c)

Figure 1: (a) The visual partition of Karate network; (b) The visual partition of College football network;(c) The visual partition of Political books network.

Model	ER model	BA model	$P\&S$ model
ω	0.15	0.24	0.45
α_C	0.95	0.92	0.85

Table 2: The comparison of ω and α_C between three different types networks

Table 2 indicates that $P\&S$ model is the most robust one, for its generation mechanism includes both the similarity of nodes and the limited growth in the network, which are not available in BA and ER models. The similarity of nodes can be reflected by clustering coefficient and average degree indirectly, but is not affected by its degree distribution. BA model is more significant than ER model, and this can be easily verified from their topology properties. These results are determined by the network building mechanism. ER network [4] is constructed by establishing a link between two isolate nodes with a probability p which is lower than 1. BA network [5] is constructed based on two properties: growth property and preferential attachment property. According to the preferential attachment mechanism, newly adding nodes are attached only preferential with older large degree nodes and this is not reasonable in many real networks. To overcome this defect, Papadopoulos et al. [6] proposed a new model which considers both popularity and similarity between newly adding node and existing ones. The degree distribution of $P\&S$ network is scale-free which is exactly the same as BA network, however, the clustering coefficient is much larger. Thus, based on the robustness measurement of $gamma_C$, $P\&S$ is more robust than the other two networks.

3 The relationships between our work and some famous concepts

As introduced by Karrer et al [7], authors showed that the robustness of community structure can be effectively quantified by measuring its robustness to small perturbations in network structure. They proposed a suitable method for perturbing networks and a measure of the resulting change in community structure, i.e. the variation of informa-

tion, to quantify the robustness. This work mainly focus on comparing the difference of information between partition in real and corresponding random network. In contrast, our method doesn't need any partition and can be quantify the robustness information directly from the topology structure.

As written by Li et al [8], the authors propose a new framework to reveal hidden properties of community structures by quantitatively analyzing the dynamics of the Potts model. Specifically, they modeled the Potts procedure of community structure detection by a Markov process, which showed that a given community is corresponding to a local uniform state. Critical topological information on multivariate spin configuration could also be inferred from the spectral significance of the Markov process, such as the optimal number of communities. By contrast, this work attempts to unveil the dynamical detail of the robustness to perturbing of community structure. The two measures both can be applied without using any particular partition algorithm, however, the objective and analysis methods between them are completely different.

References

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