Community structure detection based on Potts model and network's spectral characterization

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Abstract – The Potts model was used to uncover community structure in complex networks. However, it could not reveal much important information such as optimal number of communities and overlapping nodes hidden in networks effectively. Distinct from the previous studies, we established a new framework to study the dynamics of Potts model for community structure detection using Markov process, which had a clear mathematic explanation. Based on our framework, we showed that the local uniform behavior of spin values could naturally reveal the hierarchical community structure of a given network. Critical topological information regarding to the optimal community structure could also be inferred from spectral signatures of the Markov process. A two-stage algorithm to detect community structure is developed. The effectiveness and efficiency of the algorithm had been theoretically analyzed as well as experimentally validated.

Introduction. – Uncovering community structure [1] [2] [3] in complex networks has been studied for decades. Community structure refers to the occurrence of groups of nodes in a network that are more densely connected internally than with the rest of the network. In the early stage, these studies were restricted to the regular networks. Recently, inspired by several common characteristics of real networks [4], for example scale-free property, the majority of the studies focuses on networks with complex topologies.

Potts model has also been applied to uncover community structure in networks. Detecting community by using Potts model [5], also known as the superparamagnetic clustering method, has been a subject of intensive research since its introduction by Blatt *et al* [6]. The physical aspects of the method and its dependence on the definition of the neighbors, type of interactions, number of possible states, and size of the dataset have been well studied [7] [8] [11]. Reichardt and Bornholdt [9] introduced a spin glass Hamiltonian with a global diversity constraint to identify proper community structures in complex networks. Despite those excellent works, the relationship between dynamics of Potts model and other dynamics, such as Markov process, has not been studied. Furthermore, there are still many fundamental questions which have not yet been clearly answered. For example, can Potts model reveal critical properties of a given network such as the optimal number and stability of community structure? Is Potts model able to uncover multiscale communities in large real networks? How can Potts model recognize overlapping communities effectively?

We notice that the dynamics of Markov process can naturally reflect the intrinsic properties of spin dynamics in networks with modularity structures and exhibit local uniform behaviors. In this work, using the Potts model and spin-spin correlation, we first investigate this phenomenon, and then uncover the relation between the community structure of a network and its meta-stability of spin dynamics, and further propose spectral signatures to characterize and analyze the stability of community. For any given network, one can straightforwardly derive critical information related to its modularity, such as the stability of its community structures and the optimal number of communities from its spectral signature without us-

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ing particular algorithms. It overcomes the inefficiency of the classic methods, such as the resolution limitation of modularity Q [23] [10]. Based on the basic properties of Potts model and Markov process, we then develop a two-stage algorithm to numerically detect community structure, which is able to identify overlapping communities by associating each node with a membership vector that describes node's involvement in each community. We also demonstrate that the algorithm is scalable and effective for large scale networks.

The outline of the paper is as follows. Section 2 introduces the Potts model and the motivation of this work. In Section 3, we present a Markov stochastic model, which explains the relationship between spectral signatures and community structure. Section 4 describes our algorithm, and numerical computation for representative networks is presented in Section 5 to validate the effectiveness and efficiency of the algorithm. Section 6 concludes this paper.

Potts model and spin-spin correlation . – The Potts model is one of the most popular models in statistical mechanics [5]. It describes a system of spins that can be in q different states. Given an unweighted network with N nodes without self-loops, a spin configuration $\{S\}$ is defined by assigning each node i a spin label s_i which may take integer values $s_i = 1, ..., q$. The Hamiltonian $H(S)$ of a Potts model with this spin configuration S is given by:

$$H(S) = \sum_{\langle ij \rangle} J_{ij}(1 - \delta_{s_i s_j}), (i, j = 1, ..., N) \qquad (1)$$

where the sum is running over all neighboring nodes denoted as $\langle ij \rangle$, J_{ij} is the interaction between a pair of spins associated with the nodes i and j, and $\delta_{s_i s_j}$ is 1 if $s_i = s_j$, otherwise 0. J_{ij} is set as

$$J_{ij} = J_{ji} = \frac{1}{\langle k \rangle} \exp[-\frac{(d_{ij})^2}{2}], (i, j = 1, ..., N) \qquad (2)$$

where $\langle k \rangle$ is the average number of neighbors per node and d_{ij} is the Euclidean distance between nodes i and j. The interaction J_{ij} is a monotonous decreasing function of d_{ij} and the spins s_i and s_j tend to have the same value as d_{ij} becomes smaller.

To characterize the coherence and correlation between two spins, spin-spin correlation function C_{ij} is defined as the thermal average of $\delta_{s_i s_j}$ [6] [7] [8]:

$$C_{ij} = \langle \delta_{s_i s_j} \rangle \qquad (3)$$

It represents the probability that spin variables s_i and s_j have the same value. The measure C_{ij} takes values from the interval [0, 1], representing the continuum from no coupling to perfect accordance of spins i and j. There are two phases in a homogeneous system where J_{ij} is determined. At high temperatures, the system is in the paramagnetic phase and the spins are in disorder. $C_{ij} \approx \frac{1}{q}$ for all nodes i and j, and q is the number of possible spin values. At low temperatures, the system turns into the ferromagnetic phase and all the spins are aligned to the same direction. $C_{ij} \approx 1$ holds for nodes pair i and j.

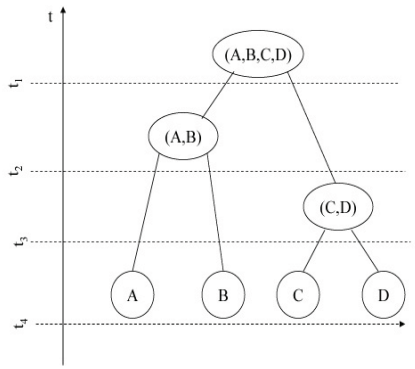


Fig. 1: Dynamics of spin values of four communities (A, B, C, D) when they go through several local uniform states to the global stable state with temperature decreasing. At temperature t_4 ($t_4 > t_3 > t_2 > t_1$, t_i denotes the temperature that i spin states exist in the system), we observe four local uniform spin state distributions corresponding to four communities. At temperature t_3, C and D mix together. At t_2, A with B mix together in terms of their hierarchical structure. Finally, at t_1, only one spin state is left in the system, in which all nodes have an identical spin distribution.

If the system is not homogeneous but has a community structure, the states are not just ferromagnetic or paramagnetic. We assume that before spins reach a globally stable state with all same value as temperature decreases, they will go through a hierarchy of local uniform states (meta-stable states) first, as shown in Fig.1. In each local uniform state, due to the dense connection, spin values of nodes within the same communities are identical and the whole system is divided into several different local regions (communities). Correspondingly, we can calculate the hitting and exiting time of each local uniform state to analyze its stability. The hitting or exiting time is the time scale that the system just enters or leaves this local uniform state, during which the spin values of nodes will stably stay on this state. We can associate the community structure with a local uniform state. For a well-formed community structure, each community should be cohesive, which means that it is easy for the nodes to hit the local uniform state. Thus, the hitting time should be early. At the same time, communities should stand clear from each other, which means it is hard for nodes to exit the local uniform state, therefore the existing time should be late. Hence, there should be a big gap between the hitting and exiting times corresponding a well-formed community structure.

Once J_{ij} has been determined, C_{ij} can be obtained by a Monte Carlo procedure. We used the Swendsen-Wang (SW) algorithm [12] because it exhibits much smaller autocorrelation times [12] than standard methods and also

provides an improved estimator of C_{ij}. We set a the initial number of possible spin values $q = N/2$ because if the number of communities is smaller than q, the excess spin states will not be populated. For a specific node, we choose a initial spin value randomly from 1 to q.

A stochastic model and spectral signatures of networks. – In this section, we will discuss the connection between the community structure of a network and the local uniform behavior of Potts model by introducing a Markov stochastic model, featured with spectral signatures for the network. Let $G = (V, E)$ denote a network, where V is the set of nodes and E is the set of edges (or links). Consider a Markov random walk process defined on G, in which a random walker freely walks from one node to another along their links. After arriving at one node, the walker will randomly select one of its neighbors and move there. Let $X = X_t, t \geq 0$, denote the walker positions, and $P\{X_t = i, 1 \leq i \leq N\}$ be the probability that the walker hits the node i after exact t steps. For $i_t \in V$, we have $P(X_t = i_t|X_0 = i_0, X_1 = i_1, ..., X_{t-1} = i_{t-1}) = P(X_t = i_t|X_{t-1} = i_{t-1})$. That is, the next state of the walker is determined only by its current state. Hence, this stochastic process is a discrete Markov chain and its state space is V. Furthermore, X_t is homogeneous because of $P(X_t = j|X_{t-1} = i) = p_{ij}$, where p_{ij} is the transition probability from nodes i to j. Markov random walk [18] has been used to find communities [19] [20]. If a graph has a strong community structure, a random walker spends a long time inside a community due to the high density of internal edges and consequent number of paths that could be followed.

To relate the Markov process with the patterns of Potts model, p_{ij} is defined as

$$p_{ij} = \frac{C_{ij}}{\sum_{j=1}^{N} C_{ij}} \tag{4}$$

where C_{ij} is the spin-spin correlation function defined in Eq.(3). Via this representation, the tools of stochastic theory and finite-state Markov processes [19] [20] can be utilized for the purposes of community detection analysis. Let P be the transition probability matrix, we have:

$$P = D_C^{-1} C \tag{5}$$

where D_C is the diagonal degree matrix of C. Let $p_{ij}^{(\tau)}$ be the probability of hitting unit j after τ steps starting from unit i, we have:

$$p_{ij}^{(\tau)} = (P^\tau)_{ij} \tag{6}$$

For this ergodic Markov process, P^τ corresponds to the probability of transitions between states over a period of τ time steps. To compute the transition matrix P^τ, the eigenvalue decomposition of P is used. If λ_k with $k = 0, ..., N-1$ denote the eigenvalues of P, and its right and left eigenvectors f_k and h_k are scaled to satisfy the orthonormality relation [20]:

$$f_k h_l = \delta_{kl} \tag{7}$$

, the spectral representation of P is given by

$$P = \sum_k \lambda_k f_k h_k \tag{8}$$

and consequently

$$P^\tau = \sum_k \lambda_k^\tau f_k h_k \tag{9}$$

We assume that eigenvalues of P are sorted such that $\lambda_0 = 1 > |\lambda_1| \geq |\lambda_2| \geq ... \geq |\lambda_{N-1}|$. The convergence of every initial distribution to the stationary distribution $P^{(0)}$ corresponds to the fact that the spin of whole system ultimately reaches exactly the same value, as temperature decreases when time goes on. This perspective belongs to a timescale $\tau \to \infty$, at which all eigenvalues λ_k^τ go to 0 except for the largest one, $\lambda_0^\tau = 1$. In the other extreme of a timescale $\tau = 0$, P^τ becomes the identity matrix. All of its columns are different, and the system disintegrates into as many spin values as the elements there are.

For the purposes of community identification, intermediate timescales of local uniform state are of interest, on which many but not all of the eigenvalues are practically zero. If we want to identify z communities, we expect to find P^τ at a timescale, the eigenvalues λ_k^τ of which may be significantly different from zero only for the range $k = 0, ..., z-1$. This is achieved by determining τ such that $|\lambda_z|^\tau \approx 0$. Using a parameter $\zeta \ll 1$ which is considered to be practical zero, we require $|\lambda_z|^\tau = \zeta$ to determine the appropriate hitting time for the whole system entering into a local uniform state with z different spin values (z-state):

$$\tau(z) = \frac{\log \zeta}{\log |\lambda_z|} \tag{10}$$

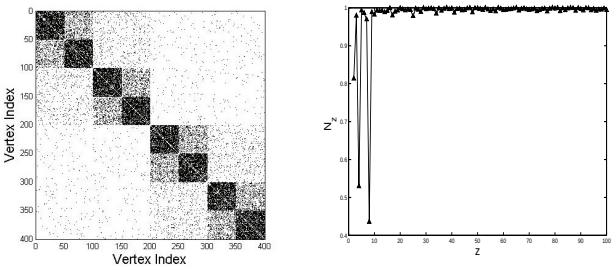
The vanishing of the smaller eigenvalues at a given timescale describes the loss of different spin states, and the removal of the structural features encoded in the corresponding weaker eigenvectors.

We define the stability of z community structure, N_z, as the ratio between the hitting time and exiting time of z-state, $\tau(z)$ and $\tau(z-1)$:

$$N_z = \frac{\log \zeta / \log |\lambda_z|}{\log \zeta / \log |\lambda_{z-1}|} = \frac{\log |\lambda_{z-1}|}{\log |\lambda_z|} \tag{11}$$

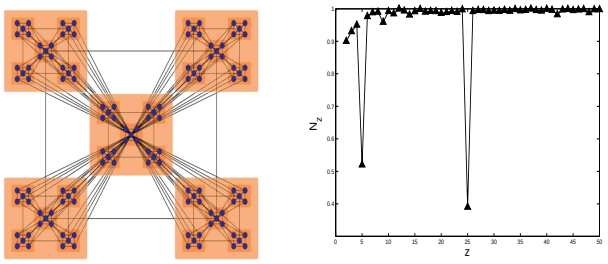
Because of $\log \zeta / \log |\lambda_z| \leq \log \zeta / \log |\lambda_{z-1}|$, it is easy to show $0 \leq N_z \leq 1$, and a smaller N_z implies a better community structure. For real networks, we can use the label of the smallest N_z to estimate the natural number of communities, *opt*, in a given network:

$$opt = \arg[\min_q(N_z)] \tag{12}$$



(a) (b)

Fig. 2: (a) A hierarchical network with 3-level community structure with 400 nodes. Due to heavy link density, it most likely contains eight small communities. Each two small communities are contained in a moderate community and finally the whole network is partitioned into two big sparse ones. (b) The stability N_z versus the number of communities z.



(a) (b)

Fig. 3: (a) Structure of $RB125$ [13] with partition of 25 dense communities and 5 sparse communities is highlighted on the original network. (b) The stability N_z versus the number of communities z.

To show that our method can discover hierarchical structures in different scales, Fig.2 and Fig.3 give two example of the multi-level community structures. In these cases, the number of N_z ($z > 1$) approaching to zero reveals the actual number of hierarchical levels hidden in a network. Furthermore, the signature of such levels can be quantified by their corresponding values of N_z.

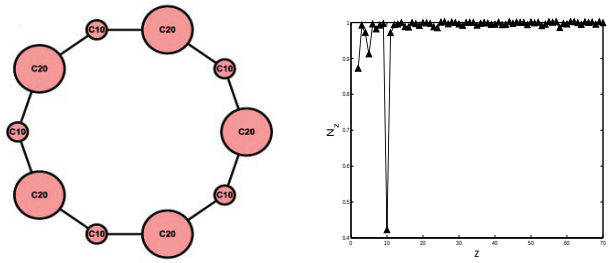
Finally, we emphasize the difference of N_z proposed in this paper and the modularity Q proposed by Newman [1] [3]. The Q is a widely used criterion for evaluating a specific partition scheme of a network, it is defined as "the fraction of edges that fall within communities, minus the expected value of the same quantity if edges fall at random without regard for the community structure" [3] [10] [21] [22]. Different partition schemes will get different Q values for the same network, and larger ones mean better partitions. While N_z tries to directly characterize and evaluate the structure property of networks based on network's spectra, rather than a specific network partition based on a predefined function. In addition, Fortunato *et al* [23] pointed out the resolution limit problem of the modularity Q, that is, there exists an intrinsic scale beyond which the smaller communities cannot be detected by maximizing the modularity. However, as shown in Fig.4, when a clique ring contains two scale cliques (i.e.,the heterogeneous community size), the intrinsic community structure can be exactly revealed by N_z.

The algorithm. – To actually perform the community detection, we propose a two-stage algorithm, which is carried out in a distributed manner. In the first stage, nodes will collaborate to find their "roles" in the community. This is a local measure of how 'well connected' each node is. Identifying the role of nodes in the community is very important to analyze the properties of the complex networks. Many ways can be used to define the "leader node", such as the nodes with largest degree or betweenness centrality. Here, we can naturally use the distribution of phase difference defined above to search the leader nodes, and the notion of "leadership" θ_i is defined as the sum of phase difference:

$$\theta_i = \sum_{j=1}^{N} J_{ij} C_{ij} = \frac{1}{\langle k \rangle} \sum_{\langle ij \rangle} \langle \delta_{s_i s_j} \rangle exp[-\frac{(d_{ij})^2}{2}] \qquad (13)$$

where J_{ij} is the interaction between a pair of spins associated with the nodes i and j, C_{ij} is the spin-spin correlation, d_{ij} is the Euclidean distance and $\langle k \rangle$ is the average number of neighbors per node. We observe that leadership θ_i is actually an quality parameter and can be used to reflect the stability of nodes: nodes with large quality parameter are well connected, and can easily reach a stable state and keep it with a stable manner. Once assigning leaderships to the nodes, we can choose z leaders of the graph with the largest leadership values. Note that in the rare cases, two or more leaders are also most influential neighbors to each other, then they are grouped together and both become leaders of a same group. For example, in a fully connected network, all of the nodes are leaders of one community, whereas for a ring network, each node is a leader.

In the second stage, we use a simple Markov dynamic system to assign each node a vector that includes compact global information on how the node is located with respect to the other nodes. Consider a graph with z leaders $l_1, l_2, ..., l_z$ and $N - z$ regular nodes, z is the optimal number of communities calculated above. Given the leaders and a arbitrary order assigned to them, we first determine the membership vectors for each regular node. We calculate the membership vector of node i by $x_i = [x_i^1, x_i^2, ..., x_i^z]$, a probability vector with length z that describes node i's involvement in each community. $x_i^k(t)$ means the kth entry of the membership vector of node i at time t. The procedure operates as follows. The membership vector of leader l_i is first assigned to be the unit vector x_{li}=1. These z vectors do not vary. For all regular nodes i, x_i^k is initialized randomly by uniform distribution on $[0, 1]$. At iteration t, each regular node i, updates its membership vector entry-wise ($i = 1, 2, ..., z$) using the



(a) (b)

Fig. 4: (a) Clique circle network as a schematic example. Each circle corresponds to a clique, whose size is marked by its label C20 (contains 20 nodes) or C10 (contains 10 nodes). (b) Stability N_z versus the number of communities q.

following dynamic system:

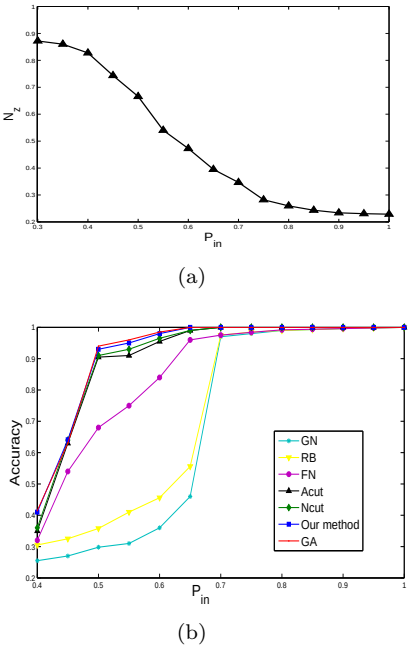
$$x_i^k(t+1) = \frac{1}{\sum_j a_{ij}} \sum_j a_{ij}(x_j^k(t)) \qquad (14)$$

where $A = (a_{ij})$ is the adjacent matrix with $a_{ij} = 1$ if there is an edge between nodes i and j, the value k indicates the k-th community.

At each time step, the membership vector of each node is updated by computing a weighted average of the membership vectors of its neighbors. We notice that Eq.(14) is equivalent to $X(t+1) = PX(t) = D^{-1}AX(t)$, where $P = D^{-1}A$ is a standard random walk transition probability matrix, D is the diagonal matrix with i-th diagonal element equal to the degree of node i. Actually, the influence of leader nodes l_k $(k = 1, 2, ..., z)$ on any regular node i, x_i^k, is the probability that a random walk starting from i hits leader l_k before it hits any other leader nodes [24]. If the underlying graph is connected, the iteration $\lim_{t\to\infty} x_i(t)$ converges to a set of unique vectors. We normalize each row of these vectors so that they can naturally be represented as the probability that a regular node belongs to the community with a given leader node in [24]. As a result, we can use the algorithm to gain membership containing global information of the whole graph.

Experiments. – In this section, we will test the performances of our algorithm. We designed and implemented two experiments for two main objectives: (1) to evaluate the accuracy of the algorithm; (2) to apply it to real large-scale networks.

Ad-Hoc network . We compared the accuracy of our algorithm with other six most well-known algorithms, including: GN algorithm [14], FN algorithm [2], two spectral methods (Ncut algorithm [15] and Acut algorithm [16]), GA algorithm [17] and RB Potts algorithm [9] for a widely used Ad-Hoc network model, which can produce a randomly synthetic network containing 4 predefined communities and each has 32 nodes. The average degree of nodes is 16, and the ratio of intra-community links is denoted as P_{in}. As P_{in} decreases, the community structures



(a)

(b)

Fig. 5: (a) $N_z(z = 4)$ values of networks versus different P_{in}. (b) Comparison of accuracy with other six algorithms.

of Ad-Hoc networks become more and more ambiguous, and correspondingly, their N_4 values climb from 0 to 1, as shown in Fig.5(a).

Communities are considered to be correctly discovered if all nodes are clustered into four original groups. Fig.5(b) presents the experimental results, in which y-axis denotes the fraction of nodes correctly clustered, and each point in curves was obtained by testing them against 200 synthetic networks shuffled from the original network. As we can see, all algorithms work well when P_{in} is larger than 0.7 ($N_4 < 0.36$) with accuracy larger than 0.95. Compared with other six algorithms, our algorithm overall outperforms other algorithms and its accuracy is only slightly worse than that of the GA in the case of $0.5 \leq P_{in} \leq 0.65$.

Scientific collaboration network . Finally we tested our algorithm on a large-scale network, the scientific collaboration network, collected by Girvan and Newman [14]. The network illustrates the research collaborations among 56276 physicists in terms of their coauthored papers posted on the Physics E-print Archive at arxiv.org. Totally, this network contains 315810 weighted edges. For visualization purpose, our algorithm outputs a transformed adjacency matrix (in which the nodes within the same communities will be arranged together) with a hierarchical community structure. From the transformed matrix of Figs.6(a), one can observe a quite strong community structure, or a group-oriented collaboration pattern, among these physicists, in which three biggest research communities are self organized regarding to three main research fields: condensed matter, high-energy physics (including theory, phenomenology and nuclear), and astro-

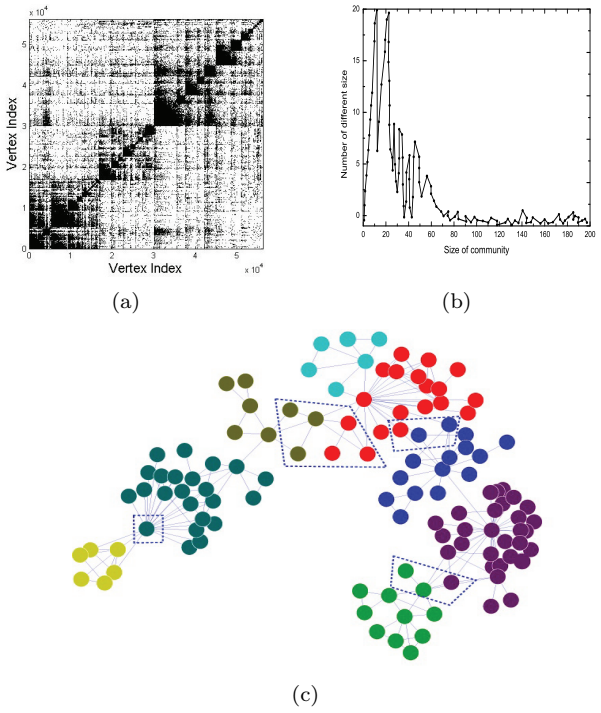


Fig. 6: (a) Transformed adjacency matrix of scientific collaboration network. (b) Distribution of community size in a linear plot. (c) Subnetwork including eight communities with 13 overlapping nodes enclosed by four rectangles.

physics. The distribution of community sizes is shown in Fig.6(b). Totally, 843 communities were detected with optimal community stability of N_z, the maximum size of those communities is 199, the minimum size is 2, and the average size is 67. Furthermore, a subnetwork including eight communities is shown in Fig.6(c) and four regions including 13 overlapping nodes are highlighted by by four rectangles in Fig.6(c) , which were detected according to membership vector. The partition is totally the same as the results of Refs. [14] and [22] which have been tested and verified. The ability to find overlapping nodes of our method is useful to reveal a natural characteristic in many real networks.

Conclusion. – In summary, we have presented a new community detection method which is able to uncover the connection between community structures and network's spectrum properties of local uniform state, based on Potts model. We demonstrate that important information related to community structures can be mined from a network's spectral signatures, such as the stability of modularity structures and the optimal number of communities. Based on theoretical analysis, we further developed a two-stage algorithm which can be implemented in a distributed manner and can identify overlapping communities, utilizing a Markov dynamical system. Its effectiveness and efficiency have been demonstrated and verified using both benchmark network and large-scale network.

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